# A computational analysis on similarities and dissimilarities of Acem Kurdi, Kurdi and Muhayyer Kurdi makams in Turkish music 

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#### Abstract

In Turkish music, it is possible to find different makams sharing the same core scale of notes. The subjects of this study are three such makams, namely Acem Küdri, Kürdi, Muhayyer Kürdi. We use computational analysis based on histograms, pattern search and dynamic time warping to reveal the similarities and dissimilarities of these three makams. On the one hand, our results show that a time independent histogram analysis is unable to properly highlight the differences between different makams. On the other hand, our study also reveals that a time dependent analysis is well suited for the identification of their distinguishing features. In particular, the application of a specialized dynamic time warping technique leads to the establishment of low correlation between these makams.


Keywords: Acem Kurdi, Kurdi, Muhayyer Kurdi makams, pattern search, seyir, dynamic time warping.
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## INTRODUCTION

The concept of makam in Turkish music has a loose association to the key signature of western music. Like the major and minor scales of western music, it is partially based on note intervals. Yet, this is quite an incomplete description, since in the construction of a makam; it is possible to find the mixed use of accidentals in the form of flats and sharps. Moreover, different makams may use the same key structure and occupy exactly the same notes range in the core region, but will be differentiated due to their constituent chords, expansion regions and particularly the melodic progression. In this context, the concept of makam is well beyond the sole definition of key signatures. Additionally, with a freedom of using accidentals in a mixed manner, it is feasible to design quite a number of makams. For instance, a recent compilation lists three hundred and fifty six makams (Academy - Makam, 2011). As a credible archiving source of Turkish music, Turkish Radio and Television (TRT) sources give the names of one hundred forty six makams (Ayangil, 2008; Bozkurt et al., 2014b). But quite a lower number is popularly used in Turkey
(Signell, 2008).
The other distinguishing features of Turkish music and the related subject of makam, particularly the common scales shared by several different makams and hence the important role of melodic progression for identification are extensively investigated in Ayangil (2008), Aybars (2008), Bartolini and Ciaccia (2005), Benetos and Holzapfel (2015), Bozkurt and Karaçalı (2015), Bozkurt (2008), Bozkurt et al. (2014a), Bozkurt et al. (2014b), Casey and Slaney (2006), Feng et al. (2010), Gedik and Bozkurt (2009), Kashima (2010), Keogh and Pazzani (2001), Li et al. (2008), Ottoman Music - Seyir (2018), Signell (2008), Stettiner et al. (1994), Tanrıkorur (1996), TRT Note Archive (2018), Ünal et al. (2014), Wollmer et al. (2009), Yaniv and Burshtein (2003). With the use of intelligent software tools, such studies have also contributed a great deal to the understanding of other aspects of makam music. The central frequencies of notes in Turkish music are slightly shifted with respect to the western counterpart except for the note A4 (Dügah). There is the use of commas in Turkish music, which do
not exactly coincide with the positions of flats and sharps of western music. Other differences and appropriateness of western music concepts and analysis to Turkish music are discussed at length in Bozkurt et al., (2014a, b), Casey and Slaney (2006) and Feng et al. (2010). From a computational point of view, such distinctions would have to be taken into account if Turkish as well as western music were to be investigated in the same study. Otherwise such details become irrelevant. In the current paper, the sole aim is Turkish music; hence the use of western notation will cause no confusion, but facilitate ease of apprehension.
To illustrate better the peculiarities of Turkish makam music, the present study examines three makams, namely Acem Kürdi, Kürdi and Muhayyer Kürdi sharing the same core region of notes (Figure 1). Our analysis is based on histogram, pattern search and dynamic time warping (DTW), where the later two will include the time progression properties of the musical pieces, called seyir.

## SCALES AND SEYIRS FOR ACEM KURDI, KURDI AND MUHAYYER KURDI MAKAMS

The constituent chords for the core and expansion regions of Acem Kürdi, Kürdi and Muhayyer Kürdi are shown in Figure 1. The notes are written with Turkish and approximated western names. As explained above, the western note names shall be used from this point onwards. In the core region of these makams, there is a nonshifted Kürdi-4 tetrachord and then there is a shifted Buselik-5 pentachord (the beginning of the original Buselik-5 pentachord is at A4, whereas here it is shifted to begin at D5, hence we call it shifted). The two chords are fused at D5. According to Figure 1, Acem Kürdi, Kürdi and Muhayyer Kürdi makams expand only toward the treble note range with the largest expansion being exhibited by Muhayyer Kürdi. As will be shown in section 4, the practical performances have even slightly larger ranges.
According to the seyir descriptions found in the literature (Academy - Makam, 2011; Gedik and Bozkurt, 2009), Acem Kürdi makam sets out its melodic journey using the shifted Buselik-5 pentachord of the core region with the beginning note most usually being F5. This is continued by expeditions through the shifted Buselik-5 pentachord and Çargah-5 pentachord of the expansion region, where there are frequent suspended cadences on F5. Then, the melody enters Kürdi-4 pentachord via C5 or A4. There, variations of the Kürdi-4 tetrachord are shown with the melody finally ending around A4.
The seyir of the Kürdi makam is said to start with the Kürdi-4 tetrachord, usually using A4 or D5. During the melodic journey through the Kürdi-4 tetrachord, frequent suspended cadences on D5 are encountered. The melody advances towards the Buselik-5 pentachord, where intermediate acts, generally involving the dominating note with longer note durations are
demonstrated. Eventually following the final take-off at D5, using the notes of the Kürdi-4 tetrachord, the melodic journey terminates around A4.
Muhayyer Kürdi makam starts the melodic journey with the Buselik-5 pentachord, usually around A5, and then steps into the upper expansion region of the shifted Hüseyni-5 pentachord. Exhibiting variations in these two pentachords, it enters the Kürdi-4 tetrachord via D5. Whilst there, suspended cadences at C5 are observed. Finally Muhayyer Kürdi makam ends around A4.

## CONSTRUCTION OF A SUITABLE DYNAMIC TIME WARPING (DTW) METHOD

DTW is a powerful approach to measure the similarities and dissimilarities between two functions, generally having different time durations. Several basic and more developed versions have been reported in the literature for applications such as music and object recognition in general (Bozkurt and Karaçalı, 2015; Bozkurt, 2008; Bozkurt et al., 2014a; Bozkurt et al., 2014b; Casey and Slaney, 2006; Feng et al., 2010; Gedik and Bozkurt, 2009; Kashima, 2010; Keogh and Pazzani, 2001; Li et al., 2008; Ottoman Music - Seyir, 2018; Signell, 2008; Stettiner et al., 1994; Tanrikorur, 1996; TRT Note Archive, 2018; Ünal et al., 2014). The usual trend is to build a distance matrix from the numeric values of each and every cross element of the two functions and then find the shortest (diagonal) path in this DTW matrix. This path is supposed to be a trace though the most similar parts of the two functions. Within the framework of the present study, it was seen that this kind of analysis did not produce meaningful results, since the melodic movement of our musical pieces comprised a variety of non-matching phases. Besides, the respective durations were relatively much longer than the cases examined in the literature. This way, whatever two functions were taken, saturation would soon occur, thus the so-called shortest path would yield no useful information about the similarity or dissimilarity of the two functions in question. Observing this, the following modified version of DTW was devised and executed.
To explain the version of DTW used in this study, we take two time functions $S_{1}$ and $S_{2}$ representing the time melodic progression of two songs such that:

$$
\begin{equation*}
S_{1}=\sum_{i_{1}=1}^{N_{1}} s_{i_{1}} \quad, \quad S_{2}=\sum_{i_{2}=1}^{N_{2}} s_{i_{2}} \tag{1}
\end{equation*}
$$

Note that in Equation 1, $i_{1}$ and $i_{2}$ imply the common time decimations for $S_{1}$ and $S_{2}$, while $s_{i_{1}}$ and $s_{i_{2}}$ are the (MIDI) decimal note values. A DTW distance matrix $D$ is constructed, whose elements are calculated from the

## Common core region



Figure 1. Scales of Acem Kürdi, Kürdi and Muhayyer Kürdi makams.
relationship:
$d_{i_{1}, i_{2}}=\left(s_{i_{1}}-s_{i_{2}}\right)^{2}$

This way, $d_{i_{1}, i_{2}}$ represents the distance measured
between the $i_{1}$ st note value of $S_{1}$ and the $i_{2}$ nd note value of $S_{2}$. Under the assumption of $N_{1} \geq N_{2}$, the (ftetorial appearance of matrix $D$ and its important properties are exhibited in Figure 2.

Regarding Figure 2, it is possible to arrive at the following deductions, particularly bearing in mind the


Figure 2. Pictorial appearance and important properties of distance matrix $D$.
above comments:

- To quantitatively assess the similarity of the time melodic progression of two musical pieces, as a better alternative to the concept of the shortest path from 1,1 to $N_{1}, N_{2}$, we may consider the region between the diagonals, 1,1 to $N_{2}, N_{2}$ and $N_{1}-N_{2}+1,1$ to $N_{1}, N_{2}$, named main diagonals (MD) region. This reasoning is due to the nearest time coincident instances of $S_{1}$ and $S_{2}$ being concentrated in this specific region. If required, the MD region may slightly be broadened, as illustrated in Figure 2, to the lower left and upper right corners of the matrix $D$ with an amount of $c N_{2}$ where $c$ is a fractional multiplier being somewhat less than unity.
- In the MD region, the degree of time coincident similarity between $S_{1}$ and $S_{2}$ is dominantly expressed by the presence of consecutive zeros along the diagonals confined therein. In this sense, single isolated zeros do not carry any meaning, and the same is true for elements of $D$ with $d_{i_{1}, i_{2}}>0$.
- The presence of consecutive zeros outside the MD region in the upper and lower parts of matrix $D$ indicate similarities in other time slices of $S_{1}$ and $S_{2}$. For instance a series of consecutive zeros along the diagonals closer to the lower left part of the matrix $D$ would mean that the parts at the end of $S_{1}$ are similar to the parts at the beginning of $S_{2}$.

Before deriving a similarity or correlation coefficient which would formally express the similarity of two musical pieces, we review the special case $S_{1}=S_{2}$. Here, we would obtain a square matrix, where the main diagonal would be filled with zeros all the way from 1,1 to $N_{1}, N_{1}$ (or $N_{2}, N_{2}$ ). If similarities also exist at different time instances, then the diagonals other than the main diagonal of $D$ would begin to possess zeros. Here the limiting situation would be when all elements of $S_{1}$ are equal. Returning to the general case $S_{1} \neq S_{2}$, we propose to define a correlation coefficient in the manner:
$\sigma_{S}=\frac{\text { Number of selected zeros in selected diagonal region of } D}{\text { Total number of elements in selected diagonal region of } D}=$

$$
\begin{equation*}
\frac{\sum_{i_{2}=1}^{N_{2}} \sum_{i_{1}=i_{2}}^{N_{1}-N_{2}+i_{2}}\left(d_{i_{1}, i_{2}}=0\right)}{N_{2}\left(N_{1}-N_{2}+1\right)} \tag{3}
\end{equation*}
$$

In Equation 3, "selected zeros" may refer to selected number of consecutive zeros along the diagonals of the selected diagonal region or any zeros in a desired arrangement, but the mathematical notation to the right accounts for the case of picking up zeros in the MD region. It is easy to see that in the limiting case of $S_{1}=S_{2}$ with all elements of $S_{1}$ being identical, $\sigma_{S}$ would simply be unity, otherwise $\sigma_{S}<1$.

## OTHER MATHEMATICAL DERIVATIONS, PRESENTATION OF OUTPUTS AND DISCUSSIONS

In this section, we provide other mathematical derivations and the results derived from the investigation of forty songs for each of Acem Kürdi, Kürdi and Muhayyer Kürdi makams. Different computational analyses were carried out on the MIDI data of these songs. In particular, a histogram analysis, a sequential pattern analysis and a DTW analysis were performed in the scope of this paper.
Regarding the histogram analysis, Figures $3 \mathrm{a}, \mathrm{b}$ and c display the histograms of notes over all songs for the different makams with the sum of values on the vertical axis normalized to one hundred percent. Here, for the reader's convenience, Turkish note names are written as well.
It is seen from Figure 3, that the note ranges spanned and the distribution of dominant notes differ somewhat among the makams. To this end, Muhayyer Kürdi has the
widest note range, while Acem Kürdi has the narrowest. This observation is partially in agreement with the general appearance in Figure 1. Figure 3 also reveals that, for all makams in question, the usage of notes is heavily confined to the core region of these makams, i.e. from A4 to A5. According to Figure 3, the most dominating notes for Acem Kürdi, Kürdi and Muhayyer Kürdi makams are F5, D5, A5, respectively. It is worth noting that such findings are in conformity with the ones reported in the literature (Academy - Makam, 2011; Akkoç, 2002; Akkoç et al., 2015; Ayangil, 2008; Aybars, 2008; Bartolini and Ciaccia, 2005; Benetos and Holzapfel, 2015). For the case of Muhayyer Kürdi, there is apparently a conflict between Figure 1 and Figure 3 c as to whether the upper expansion region should employ B5 or Bb5. According to Figure 1, the theoretical choice seems to be B5 (or B5 with comma1 to be precise), but in practice we have found that this is mostly converted into Bb 5 as illustrated in Figure 3c.


Figure 3a. Histogram on usage of notes in forty Acem Kurdi songs.

We should remind the information contained in Figure 3 is time independent, hence providing no clues about the seyir of the studied makams. It is possible however to acquire from this static data, pair wise cross correlation characteristics of the makams under study. For this, we stipulate that the songs of a particular makam, say Acem Kürdi, are appended to the end of each other to form a long time function $S_{A}$ such that:
$S_{A}=\sum_{m=1}^{M_{A}} \sum_{i_{m}=1}^{N_{m}} s_{i_{m}}$

Where $M_{A}$ is the total number of songs in that makam and $N_{m}$ is the specific length of each song in units of the common time decimation. By applying histogram operation denoted by the operator $\mathbf{H}$ to the time function $S_{A}$, we get:
$\mathbf{H}\left[S_{A}\right]=P_{A}=\sum_{i=A_{\text {min }}}^{A_{\text {max }}}{ }_{A} p_{i}$
Where $P_{A}$ is the probability density function for note usage in Acem Kurdi songs prior to normalization, thus its


Figure 3b. Histogram on usage of notes in forty Kurdi songs.


Figure 3c. Histogram on usage of notes in forty Muhayyer Kurdi songs.
normalized version will generate the plot in Figure 3a. It is clear from Figure 3, for other makams, $A_{\text {min }}$ and $A_{\text {max }}$, which stand for the lowest and the highest note in $P_{A}$, will be different. To enable the definition of a cross correlation coefficient, the horizontal axis for different histograms must be aligned to span the same note range. This in turn calls for specifying two further parameters $C_{\text {min }}$ and $C_{\text {max }}$ which will be given by:

$$
\begin{align*}
& C_{\min }=\min \left(A_{\min }, K_{\min }, M_{\min }\right) \\
& C_{\max }=\max \left(A_{\max }, K_{\max }, M_{\max }\right) \tag{6}
\end{align*}
$$

Where $A_{\text {min }}, K_{\text {min }}, M_{\text {min }}$ and $A_{\text {max }}, K_{\text {max }}, M_{\text {max }}$ will be the minimum and maximum notes respectively in the histograms of Acem Kürdi, Kürdi and Muhayyer Kürdi makams.

As a degree of similarity between two makams, say Acem Kürdi and Kürdi, the cross correlation coefficient is:

$$
\begin{equation*}
\rho_{A K}=\frac{\sigma_{A K}}{\sigma_{A} \sigma_{K}} \tag{7}
\end{equation*}
$$

$\sigma \mathrm{s}$ in Equation 7 refers to the standard deviations in $P_{A}$

$$
\begin{align*}
& \text { and } P_{K} \text {. They are hence defined as } \\
& \sigma_{A K}=\sum_{i=C_{\text {min }}}^{C_{\text {max }}}\left({ }_{A} p_{i}-m_{A}\right)\left({ }_{K} p_{i}-m_{K}\right), \\
& \sigma_{A}=\sqrt{\sum_{i=C_{\text {min }}}^{C_{\text {max }}}\left({ }_{A} p_{i}-m_{A}\right)^{2}}, \sigma_{K}=\sqrt{\sum_{i=C_{\text {min }}}\left({ }_{K} p_{i}-m_{K}\right)^{2}} \text { (8) } \tag{8}
\end{align*}
$$

Where $m s$ are the means given by:

$$
\begin{equation*}
m_{A}=\frac{\sum_{i=C_{\min }}^{C_{\text {max }}}{ }_{A}}{\sum_{m=1}^{M_{A}} N_{m}} \quad, \quad m_{K}=\frac{\sum_{i=C_{\text {min }}}^{C_{\text {max }}}{ }_{K} p_{i}}{\sum_{m=1}^{M_{K}} N_{m}} \tag{9}
\end{equation*}
$$

Based on the above formulations, after defining the cross correlation coefficients between all makams and making the related computations, we obtain the results listed in Table 1.

Table 1 indicates that the strongest correlation is between Acem Kürdi and Kürdi, and there is medium correlation between Acem Kürdi and Muhayyer Kürdi and least correlation exits between Kürdi and Muhayyer Kürdi.

Next we turn to the presentation of data that includes the timing element as well. In this sense, Table 2 shows the results of searches for patterns of notes that occur at least twenty times in the analysed songs of the respective makam. It is known that the makams in Turkish music are based on tetrachords and pentachords (Kashima, 2010; Stettiner et al., 1994). To this end, Table 2 was prepared by separately searching for patterns of maximum lengths of four and five and remaining shorter ones in forty songs of each makam. It was found that in such a search, the remaining shorter patterns of length two and three would constitute the minority, and hence were eliminated from the lists of Table 2. Their actual numbers and respective occurrences are noted at the bottom of each individual part of the tabulation. Table 2 is organized in the form of five columns respectively displaying the pattern itself, its total number of occurrences, number of songs (out of the forty) where this particular pattern is found, peak time location of this pattern within the relative durations of songs and finally relative time location of this pattern in the sample seyir of the makam examined. It should be pointed out that in column four, the relative time locations

Table 1. Cross correlation coefficients of the three makams based on the histogram data of Figure 3.

| $\rho_{A K}$ | $\rho_{A M}$ | $\rho_{K M}$ |
| :---: | :---: | :---: |
| 0.97 | 0.94 | 0.91 |

actually map to a distribution whose horizontal axis may extend from zero to unity with discontinuities in-between. But in this column only the peak of this distribution is quoted. The relevant seyirs were obtained from (Academy - Makam, 2011; Gedik and Bozkurt, 2009). Being a sample representative, the seyirs cannot inevitably accommodate all possible patterns found in actual performances, a fact clearly reflected in Table 2.
It is interesting to note that for the cases of pattern lengths of five, Table 2 confirms once again that, the most frequently used note is F5 for Acem Kürdi, D5 for Kürdi and A5 for Muhayyer Kürdi. Now we turn to more striking features of Table 2, that is, the associated seyir data embedded therein. According to the description given in section 2, Acem Kürdi makam is assumed to start using the shifted Buselik-5 pentachord usually around F5. Looking at the parts 2 a and 2 b of Table 2, we see that the pattern of length four in row 3 of Table 2a, which is G5 F5 E5 F5 and the pattern of length five in row 12 of Table 2 b , which is A5 G5 F5 E5 F5, are the representative of this statement, since the individual notes of these two (overlapping) patterns are all members of the shifted Buselik-5 pentachord. Moreover from columns four and five, it is seen that these two patterns are placed at the very beginning of both the songs of the makam and the seyir. Although none of these patterns start with F5, our additional analysis showed that F5 was indeed the most dominating starting note, being common to fifteen songs out of the total of forty. Again, in line with the description given above for the Acem Kürdi makam, after the beginning, there should be expeditions within the shifted Buselik-5 pentachord and even beyond. The patterns, D5 E5 F5 G5 and G5 F5 E5 D5 in the rows 1 and 12 of Table 2a, the patterns Bb5 A4 G5 F5 E5 and G5 A5 G5 F5 E5 in the rows 2 and 20 of Table 2 b are such examples. The transitions into the Kürdi-4 tetrachord are accomplished by the patterns C5 Bb4 A4 Bb4 in row 13 of Table 2a and the patterns D5 C5 Bb4 A4 Bb4 and G5 F5 E5 D5 C5 in the rows 9 and 17 of Table 2b. Finally, the ending towards A4 is supplied by the inverted Kürdi-4 tetrachord pattern D5 C5 Bb4 A4 in row 18 of Table 2a.
Similar seyir analogies can be envisaged for Kürdi and Muhayyer Kürdi makams. In this context, row 1 of both Table 2c and 2d, which consecutively cover the complete Kürdi-4 tetrachord and even one additional note from the shifted Buselik-5 pentachord, correspond to the start of the Kürdi makam using A4. The patterns on row 14 of Table 2c and in row 11 of Table 2d point to the existence of expeditions within the Kürdi-4 tetrachord. Attempts to

Table 2a. Patterns of length four in Acem Kurdi songs.

| Row No. | Patterns of length four for Acem Kurdi makam |  |  |  | Total number of occurrences | Number of songs pattern found | Relative peak time location in songs | Relative time location in seyir |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | D5 | E5 | F5 | G5 * | 78 | 26 | 0.05 | 0.33 |
| 2 | Bb5 | A5 | Bb5 | A5 | 28 | 10 | 0.80 |  |
| 3 | G5 | F5 | E5 | F5 * | 161 | 32 | 0.06 | 0.11 |
| 4 | C5 | E5 | D5 | C5 | 47 | 14 | 0.08 |  |
| 5 | Bb5 | A5 | G5 | F5 | 82 | 24 | 0.63 |  |
| 6 | E5 | D5 | E5 | D5 * | 87 | 23 | 0.06 | 0.22 |
| 7 | C5 | D5 | C5 | D5 | 39 | 16 | 0.56 |  |
| 8 | C5 | Bb4 | C5 | Bb4 | 27 | 10 | 0.34 |  |
| 9 | G5 | F5 | G5 | A5 | 27 | 15 | 0.51 |  |
| 10 | D5 | C5 | D5 | E5 * | 56 | 19 | 0.26 | 0.28 |
| 11 | D5 | E5 | F5 | E5 | 77 | 24 | 0.16 |  |
| 12 | G5 | F5 | E5 | D5 * | 98 | 26 | 0.45 | 0.61 |
| 13 | C5 | Bb4 | A4 | Bb4 * | 107 | 31 | 0.57 | 0.76 |
| 14 | C5 | Bb4 | A4 | C5 | 33 | 16 | 0.84 |  |
| 15 | C5 | Bb4 | A4 | E5 | 26 | 15 | 0.13 |  |
| 16 | F5 | G5 | A5 | G5 * | 21 | 11 | 0.53 | 0.37 |
| 17 | D5 | C5 | Bb4 | C5 | 28 | 10 | 0.95 |  |
| 18 | D5 | C5 | Bb4 | A4 * | 100 | 30 | 0.94 | 0.74 |
| 19 | Bb4 | A4 | G4 | A4 | 21 | 7 | 0.61 |  |
| 20 | F5 | E5 | F5 | E5 | 31 | 15 | 0.33 |  |
| 21 | G4 | A4 | Bb4 | C5 | 23 | 5 | 0.75 |  |
| 22 | F5 | E5 | G5 | F5 * | 32 | 15 | 0.65 | 0.46 |
| 23 | A4 | C5 | Bb4 | A4 | 22 | 10 | 0.95 |  |
| 24 | F5 | E5 | D5 | C5 * | 36 | 17 | 0.93 | 0.70 |

One remaining pattern of length three with a total of 23 occurrences
Two remaining patterns of length two with a total of 137 occurrences
Normalized difference over all songs $=-0.95$
Normalized difference over all patterns of length four $=-0.99$.

Table 2b. Patterns of length five in Acem Kurdi songs.

| Row <br> No. | Patterns of length five for Acem Kürdi makam |  |  |  |  | Total number of occurrences | Number of songs pattern found | Relative peak time location in songs | Relative time location in seyir |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | D5 | E5 | F5 | G5 | A5 * | 40 | 13 | 0.35 | 0.33 |
| 2 | Bb5 | A5 | G5 | F5 | E5 | 83 | 23 | 0.07 |  |
| 3 | F5 | C5 | D5 | E5 | F5 | 25 | 13 | 0.37 |  |
| 4 | C5 | E5 | D5 | C5 | D5 | 26 | 10 | 0.08 |  |
| 5 | A4 | Bb4 | C5 | D5 | E5 | 20 | 11 | 0.12 |  |
| 6 | D5 | E5 | D5 | C5 | D5 * | 21 | 11 | 0.07 | 0.24 |
| 7 | D5 | C5 | Bb4 | C5 | Bb4 | 25 | 10 | 0.34 |  |
| 8 | F5 | E5 | G5 | F5 | E5 | 37 | 14 | 0.82 |  |
| 9 | D5 | C5 | Bb4 | A4 | Bb4 * | 79 | 28 | 0.45 | 0.74 |
| 10 | C5 | Bb4 | A4 | E5 | F5 | 24 | 14 | 0.13 |  |
| 11 | E5 | D5 | C5 | Bb4 | A4 * | 31 | 16 | 0.11 | 0.72 |
| 12 | A5 | G5 | F5 | E5 | F5 * | 55 | 17 | 0.06 | 0.09 |
| 13 | D5 | E5 | F5 | E5 | D5 | 37 | 15 | 0.92 |  |
| 14 | F5 | E5 | F5 | G5 | F5 | 35 | 14 | 0.05 |  |

Table 2b. Continues.

| 15 | E5 | F5 | G5 | F5 | E5 | 28 | 14 | 0.05 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 16 | F5 | A5 | G5 | F5 | E5 * | 23 | 10 | 0.05 | 0.07 |
| 17 | G5 | F5 | E5 | D5 | C5 | 41 | 13 | 0.37 |  |
| 18 | D5 | C5 | Bb4 | A4 | G4 | 32 | 10 | 0.46 |  |
| 19 | A4 | Bb4 | C5 | D5 | C5 | 20 | 7 | 0.93 |  |
| 20 | G5 | A5 | G5 | F5 | E5 * | 21 | 8 | 0.74 | 0.39 |

Two remaining patterns of length two with a total of 47 occurrences
Normalized difference over all patterns of length five $=-1.78$.

Table 2c. Patterns of length four in Kurdi songs.

| Row No. | Patterns of length four for Kurdi Makam |  |  |  | Total number of occurrences | Number of songs pattern found | Relative peak time location in songs | Relative time location in seyir |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | A4 | Bb4 | C5 | D5 * | 156 | 26 | 0.15 | 0.02 |
| 2 | E5 | G5 | F5 | E5 | 71 | 18 | 0.92 |  |
| 3 | D5 | E5 | C5 | D5 | 48 | 13 | 0.05 |  |
| 4 | C5 | Bb4 | C5 | A4 | 31 | 7 | 0.06 |  |
| 5 | D5 | E5 | F5 | G5 * | 62 | 14 | 0.05 | 0.24 |
| 6 | E5 | D5 | C5 | D5 * | 143 | 29 | 0.26 | 0.83 |
| 7 | E5 | D5 | E5 | F5 | 77 | 20 | 0.15 |  |
| 8 | C5 | D5 | E5 | D5 | 35 | 12 | 0.25 |  |
| 9 | C5 | Bb4 | C5 | D5 | 40 | 11 | 0.15 |  |
| 10 | C5 | D5 | C5 | Bb4 * | 47 | 11 | 0.35 | 0.88 |
| 11 | D5 | C5 | Bb4 | A4 * | 153 | 25 | 0.94 | 0.90 |
| 12 | C5 | Bb4 | A4 | G4 | 36 | 12 | 0.27 |  |
| 13 | A4 | G4 | A4 | Bb4 | 35 | 7 | 0.91 |  |
| 14 | Bb4 | C5 | Bb4 | A4 | 67 | 16 | 0.29 |  |
| 15 | E5 | F5 | E5 | D5* | 42 | 14 | 0.24 | 0.37 |
| 16 | A5 | Bb5 | A5 | G5 | 85 | 25 | 0.44 |  |
| 17 | F5 | E5 | D5 | C5 * | 68 | 19 | 0.07 | 0.80 |
| 18 | E5 | F5 | G5 | A5 | 42 | 17 | 0.46 |  |
| 19 | E5 | D5 | C5 | Bb4 | 21 | 7 | 0.63 |  |
| 20 | G5 | F5 | E5 | D5* | 59 | 16 | 0.94 | 0.49 |
| 21 | A5 | C6 | Bb5 | A5 | 30 | 15 | 0.47 |  |
| 22 | G5 | F5 | E5 | F5 * | 20 | 7 | 0.08 | 0.32 |
| 23 | E5 | F5 | E5 | F5 | 28 | 7 | 0.32 |  |
| 24 | C5 | Bb4 | A4 | Bb4 | 33 | 9 | 0.10 |  |
| 25 | G5 | F5 | G5 | A5 | 26 | 7 | 0.54 |  |
| 26 | E5 | C5 | Bb4 | A4 * | 27 | 6 | 0.16 | 0.12 |
| 27 | C5 | A4 | Bb4 | A4 * | 22 | 8 | 0.27 | 0.68 |

Seven remaining patterns of length two with a total of 200 occurrences
Normalized difference over all songs $=0.0032$
Normalized difference over all patterns of length four $=-0.72$.
ascend towards the shifted Buselik-5 pentachord are apparent from the patterns on rows 5, 7, 15 and 16 of Table 2c, while row 2 of Table 2d is an example of an expedition within the shifted Buselik-5 pentachord. Descend from the shifted Buselik-5 pentachord can be detected from the rows 2, 6, 10, 17 and 20 of Table 2c
and from row 7 of Table 2d. The ending of the Kürdi makam is finally achieved with the reversed Kürdi-4 tetrachord on row 11 of Table 2c.
Judging from the patterns listed in Table $2 e$ and 2 f , there is no clear indication about the start and end notes and sequences of Muhayyer Kürdi songs. From our

Table 2d. Patterns of length five in Kürdi songs.

| $\begin{gathered} \text { Row } \\ \text { No } \\ \hline \end{gathered}$ | Patterns of Makam |  | length | five for | Kurdi | Total number of occurrences | Number of songs pattern found | Relative peak time location in songs | Relative time location in seyir |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | A4 | Bb4 | C5 | D5 | E5 * | 67 | 21 | 0.05 | 0.02 |
| 2 | G5 | F5 | E5 | D5 | E5 * | 88 | 21 | 0.72 | 0.49 |
| 3 | C5 | D5 | Bb4 | A4 | Bb4 | 25 | 9 | 0.83 |  |
| 4 | A4 | Bb4 | C5 | D5 | C5 | 25 | 9 | 0.09 |  |
| 5 | D5 | E5 | D5 | C5 | D5 | 44 | 14 | 0.82 |  |
| 6 | E5 | F5 | E5 | D5 | E5 | 28 | 12 | 0.06 |  |
| 7 | E5 | D5 | C5 | D5 | C5 * | 66 | 17 | 0.94 | 0.83 |
| 8 | D5 | E5 | D5 | C5 | Bb4 | 24 | 7 | 0.07 |  |
| 9 | C5 | D5 | C5 | Bb4 | A4 * | 34 | 11 | 0.26 | 0.87 |
| 10 | F5 | E5 | D5 | C5 | Bb4 | 81 | 19 | 0.94 |  |
| 11 | Bb4 | A4 | Bb4 | C5 | Bb4 | 40 | 9 | 0.57 |  |
| 12 | A4 | Bb4 | C5 | Bb4 | A4 | 27 | 9 | 0.22 |  |
| 13 | G5 | A5 | G5 | F5 | E5 | 20 | 9 | 0.56 |  |
| 14 | D5 | C5 | Bb4 | A4 | D5 | 23 | 7 | 0.26 |  |
| 15 | A5 | G5 | A5 | Bb5 | A5 | 27 | 12 | 0.60 |  |
| 16 | G5 | F5 | E5 | D5 | C5 | 35 | 10 | 0.44 |  |

Two remaining patterns of length two with a total of 63 occurrences Normalized difference over all patterns of length five $=-1.54$.

Table 2e. Patterns of length four in Muhayyer Kurdi songs.

| $\begin{gathered} \text { Row } \\ \text { No } \end{gathered}$ | Patterns o Muhayyer |  | ngth <br> maka | four for | Total number of occurrences | Number of songs pattern found | Relative peak time location in songs | Relative time location in seyir |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | A4 | C5 | Bb4 | A4 | 30 | 11 | 0.65 |  |
| 2 | C5 | Bb4 | A4 | Bb4 * | 111 | 25 | 0.94 | 0.68 |
| 3 | C5 | D5 | E5 | F5 * | 67 | 18 | 0.65 | 0.77 |
| 4 | G5 | Bb5 | A5 | G5 | 22 | 9 | 0.35 |  |
| 5 | C6 | Bb5 | A5 | G5 * | 67 | 17 | 0.06 | 0.20 |
| 6 | A5 | G5 | A5 | G5 * | 53 | 14 | 0.26 | 0.23 |
| 7 | F5 | E5 | F5 | E5 | 57 | 17 | 0.54 |  |
| 8 | D5 | F5 | E5 | D5 | 33 | 10 | 0.82 |  |
| 9 | E5 | C5 | D5 | Bb4 | 22 | 8 | 0.16 |  |
| 10 | F5 | A5 | G5 | F5 * | 32 | 13 | 0.15 | 0.35 |
| 11 | A5 | G5 | F5 | E5 * | 125 | 29 | 0.64 | 0.26 |
| 12 | D5 | E5 | C5 | Bb4 | 25 | 5 | 0.69 |  |
| 13 | D5 | C5 | D5 | E5 * | 52 | 15 | 0.93 | 0.60 |
| 14 | C6 | Bb5 | A5 | Bb5 * | 83 | 19 | 0.83 | 0.14 |
| 15 | D5 | E5 | C5 | D5 * | 22 | 6 | 0.63 | 0.43 |
| 16 | E5 | D5 | C5 | D5 * | 48 | 15 | 0.61 | 0.58 |
| 17 | D5 | E5 | F5 | E5 * | 27 | 11 | 0.07 | 0.78 |
| 18 | A4 | Bb4 | C5 | D5 * | 38 | 11 | 0.73 | 0.74 |
| 19 | A4 | G4 | A4 | Bb4 | 25 | 9 | 0.94 |  |
| 20 | F5 | G5 | A5 | Bb5 | 28 | 15 | 0.54 |  |
| 21 | A5 | Bb5 | A5 | G5 | 66 | 21 | 0.24 |  |
| 22 | E5 | D5 | C5 | Bb4 * | 66 | 17 | 0.10 | 0.65 |
| 23 | A5 | Bb5 | A5 | Bb5 * | 30 | 10 | 0.05 | 0.06 |
| 24 | A5 | Bb5 | G5 | A5 | 31 | 11 | 0.42 |  |
| 25 | F5 | E5 | D5 | E5 * | 25 | 11 | 0.09 | 0.29 |
| 26 | A4 | Bb4 | A4 | Bb4 * | 31 | 5 | 0.05 | 0.71 |

Table 2e. Continues.

| 27 | A5 | G5 | A5 | Bb5 ${ }^{*}$ | 20 | 7 | 0.30 | 0.03 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 28 | A5 | E5 | F5 | G5 | 25 | 6 | 0.05 |  |
| 29 | D5 | C5 | Bb4 | A4 ${ }^{*}$ | 24 | 8 | 0.94 | 0.66 |

Five remaining patterns of length two with a total of 122 occurrences
Normalized difference over all songs $=-0.75$
Normalized difference over all patterns of length four $=-1.16$.

Table 2f. Patterns of length five in Muhayyer Kurdi songs.

| $\begin{gathered} \text { Row } \\ \text { No } \end{gathered}$ | Patterns of length five for Muhayyer Kurdi makam |  |  |  |  | Total number of occurrences | Number of songs pattern found | Relative peak time location in songs | Relative time location in seyir |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Bb4 | A4 | Bb4 | C5 | D5 * | 74 | 19 | 0.94 | 0.72 |
| 2 | A5 | G5 | A5 | G5 | F5 * | 38 | 13 | 0.07 | 0.23 |
| 3 | A5 | G5 | A5 | Bb5 | A5 * | 41 | 16 | 0.32 | 0.03 |
| 4 | A5 | G5 | F5 | E5 | D5 * | 93 | 23 | 0.92 | 0.26 |
| 5 | F5 | E5 | D5 | C5 | D5 * | 43 | 15 | 0.88 | 0.57 |
| 6 | D6 | C6 | Bb5 | A5 | Bb5 * | 22 | 8 | 0.46 | 0.12 |
| 7 | Bb5 | A5 | C6 | Bb5 | A5 | 20 | 8 | 0.87 |  |
| 8 | F5 | E5 | D5 | C5 | E5 | 20 | 9 | 0.35 |  |
| 9 | A5 | G5 | F5 | E5 | F5 | 32 | 12 | 0.26 |  |
| 10 | E5 | D5 | C5 | D5 | C5 | 27 | 10 | 0.07 |  |
| 11 | C5 | Bb4 | A4 | Bb4 | A4 * | 32 | 12 | 0.58 | 0.68 |
| 12 | E5 | F5 | G5 | A5 | Bb5 | 28 | 13 | 0.14 |  |
| 13 | A5 | Bb5 | C6 | Bb5 | A5 * | 20 | 7 | 0.26 | 0.17 |
| 14 | E5 | F5 | E5 | D5 | E5 | 22 | 10 | 0.35 |  |
| 15 | C6 | Bb5 | A5 | Bb5 | A5 | 30 | 9 | 0.06 |  |
| 16 | E5 | D5 | C5 | Bb4 | A4 * | 22 | 9 | 0.21 | 0.65 |

No remaining patterns
Normalized difference over all patterns of length five $=-2.08$.
computations however, it is seen that, in majority, Muhayyer Kürdi songs start and end as mentioned in the theoretical seyir description, that is, seventeen songs start with A5, and twenty three songs end at A4. Intermediate parts of the seyir can be summarized as follows. Early expeditions within the shifted Buselik-5 pentachord are denoted by rows 6,11 of Table $2 e$ and row 4 of Table 2 f. Row 5 of Table $2 e$ and row 4 of Table $2 f$ are signs of expansion into the shifted Hüseyni-5 pentachord and even beyond. Descend into the Kürdi-4 tetrachord is expressed by the rows 3,16 and 22 of Table 2e. Final expeditions within the Kürdi-4 tetrachord before ending are encountered in row 2 in Table $2 e$ and row 1 in Table 2f.
At the bottom of each sub-table of Table 2, the direction of each makam over the total of forty songs and also over the patterns of length four and five (by also taking into account the number of occurrences) is calculated by summing the difference of (MIDI) consecutive note values and dividing the total by the number of notes in summation. As seen, all calculations except the one taken over all songs of the Kürdi makam have led to
negative numbers, meaning that all three makams may be descending. Considering that the seyir of the Kürdi makam starts around the Kürdi-4 tetrachord and ends with the same chord, it is more reasonable to trust the value (i.e. 0.0032 ) obtained from all songs, which allows the Kürdi makam to be ascending as well as descending.
If we ignore the matter of time locations, Table 2 points to the presence of common patterns across makams. Rows 15, 7, 6 of Table 2d and rows 3, 10, 14 of Table $2 f$ are such examples. To quantify the cross correlation that exists in this exemplified case, we sum the common pattern related occurrences from column 2 of Tables 2d and $2 f$ and divide it by the total number of occurrences from columns 2 of the same tables. By letting the mathematical symbol of thus defined cross correlation coefficient be ${ }_{T 5} \sigma_{K M}$, and then calculating and setting the notation for the others in a similar way, we arrive at the data in Table 3.
Table 3 shows that apart from the case of cross correlation between Kürdi and Muhayyer Kürdi makams, the cross correlation for patterns of length four seems to

Table 3. Cross correlation coefficients of the three makams for patterns of length four and five.

| ${ }_{T 4} \sigma_{A K}$ | ${ }_{T 5} \sigma_{A K}$ | ${ }_{T 4} \sigma_{A M}$ | ${ }_{T 5} \sigma_{A M}$ | ${ }_{T 4} \sigma_{K M}$ | ${ }_{T 5} \sigma_{K M}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.37 | 0.20 | 0.39 | 0.23 | 0.10 | 0.17 |

be more than the one for patterns of length five. On the other hand, from the numeric values in Table 3, another fact emerges. That is, in comparison to the values listed in Table 1, much weaker correlation or more differentiation can be found between makams, if patterns are considered.
Finally we report the findings resulting from the application of the revised DTW algorithm introduced in section 3. This way, we match common patterns with more accurate temporal location information. Stemming from Eq. (3) and confined to MD region, a new autocorrelation coefficient embracing all Acem Kürdi songs and expressing the similarity of patterns of length four over all these songs can be formulated as follows:
$D T 4 \sigma_{A A}=\frac{\sum_{m_{1}=1}^{M_{A}} \sum_{m_{2}=1}^{M_{A}} \sum_{i_{2}=1}^{N_{m_{2}}} \sum_{i_{1}=i_{2}}^{N_{m_{1}}-N_{m_{2}}+i_{2}} \frac{{ }_{2 T 4}\left(d_{i_{m_{1}}, i_{m_{2}}}=0\right)}{N_{m_{2}}\left(N_{m_{1}}-N_{m_{2}}+1\right)}}{M_{A}^{2}}$

Benefiting from Equation 10, the remaining auto and cross correlation coefficients can be drawn up for patterns of length five and other makams. Another option covers the patterns of length both four and five simultaneously. The results obtained after having taken into account all these possibilities are given in Table 4.
The first two sub-tables in Table 4 list the results of separate runs for patterns of length four and five, whereas the last sub-table displays the results achieved when patterns of both lengths are sought in the MD regions. The most striking message conveyed in Table 4 is that it is possible to establish differentiation between makams of Acem Kürdi, Kürdi and Muhayyer Kürdi via seeking similarity in the patterns of length five in the most time coincident region of DTW matrices, that is, MD regions. That is because here the ratio of auto correlation coefficients to cross correlation coefficients can reach as high as six, as can be estimated from the numeric values of the middle sub-table of Table 4.

Table 4. Auto correlation and cross correlation coefficients of patterns of lengths four and five for all makams obtained from the MD regions of the DTW matrices.

| ${ }_{D T 4} \sigma_{A A}$ | ${ }_{D T 4} \sigma_{K K}$ | ${ }_{D T 4} \sigma_{M M}$ | ${ }_{D T 4} \sigma_{A K}$ | ${ }_{D T 4} \sigma_{A M}$ | ${ }_{D T 4} \sigma_{K M}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0388 | 0.0380 | 0.0356 | 0.0135 | 0.0119 | 0.0114 |
| ${ }_{D T 5} \sigma_{A A}$ | ${ }_{D T 5} \sigma_{K K}$ | ${ }_{D T 5} \sigma_{M M}$ | ${ }_{D T 5} \sigma_{A K}$ | ${ }_{D T 5} \sigma_{A M}$ | ${ }_{D T 5} \sigma_{K M}$ |
| 0.0310 | 0.0305 | 0.0297 | 0.0061 | 0.0053 | 0.0049 |
| ${ }_{D T 45} \sigma_{\text {AA }}$ | ${ }_{D T 45} \sigma_{K K}$ | ${ }_{D T 45} \sigma_{M M}$ | ${ }_{D T 45} \sigma_{A K}$ | ${ }_{D T 45} \sigma_{\text {AM }}$ | ${ }_{D T 45} \sigma_{K M}$ |
| 0.0399 | 0.0390 | 0.0365 | 0.0145 | 0.0128 | 0.0122 |

## CONCLUSION

In this study, we have examined three very closely related makams of Turkish music, namely Acem Kürdi, Kürdi and Muhayyer Kürdi. The theoretical core note scale regions of these makams are identical with upper expansion regions showing slight differences. Our analysis shows that these similarities are clearly reflected into the numeric values of cross correlation coefficients derived from static, that is, time independent data. For instance it is seen that the cross correlation in this case can be as large as 0.97 . If the timing element is
introduced into the equations however, the picture changes drastically. In this case, the cross correlation values drop and additionally time dependent analysis also unveils where the distinguishing features of the three makams exactly lie. In this respect, patterns with lengths of four and five notes are searched and displayed along with their associated pattern counts and time locations. From these tabulations, it is seen that the songs of the three makams generally follow their predefined seyir flow along the time axis. It is further detected from these tables that common patterns among different makams constitute the minority. Furthermore, via utilizing the
facilities of the dynamic time warping method and its main diagonals region approach, a time correlated analysis is conducted. As a prominent result, it is seen that auto correlation and cross correlation coefficients obtained by such analysis can be a quite useful instrument to assess the degrees of similarity and dissimilarity between the studied makams.

## Conflict of interests

The authors declare that they have no conflict of interests.

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