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Modelling calculus readiness trajectories of pre-service teachers using a Markov chain approach

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ABSTRACT

This study investigated the calculus-readiness trajectories of pre-service teachers in Ghana using a longitudinal quantitative design and a discrete-time Markov chain framework. The participants were 363 pre-service teachers selected from an accessible population of 650 across five purposively selected Colleges of Education, with one college drawn from each of the five zones in which the colleges had been grouped. Readiness was assessed at baseline, mid-semester, and end-of-semester, and scores were converted to a common 30-point scale before classification into four readiness states: Unprepared, Partially Prepared, Prepared, and Highly Prepared. Transition count matrices, transition probability matrices, selected confidence intervals, a homogeneity check and a pooled steady-state distribution were estimated. The baseline results showed substantial underpreparedness, with most students entering calculus below the desired readiness level and no student classified as Highly Prepared. Transitions in the first half of the semester showed strong upward movement from lower readiness states, whereas later transitions were more variable, with both improvement and regression observed. The homogeneity check indicated that the two observed transition periods were not identical; therefore, the pooled steady-state distribution was interpreted cautiously as a descriptive projection under the observed transition dynamics. The findings show that calculus readiness among pre-service teachers is not fixed but develops unevenly across instructional time. Continuous diagnostics, bridging support, conceptual instruction and sustained formative feedback are therefore necessary in Colleges of Education.

Keywords: Calculus readiness; pre-service teachers; Markov chain; Colleges of Education.

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INTRODUCTION

Calculus is central to postsecondary mathematics education and to several science, technology, engineering and mathematics pathways. It is also a frequent source of difficulty for students whose prior preparation in algebra, functions, trigonometry and mathematical reasoning is weak. Without firm command of these prerequisite ideas, learners may treat calculus as a collection of disconnected procedures rather than as a coherent body of concepts involving change, variation and accumulation (Carlson et al., 2015; Thompson and Harel, 2021). For pre-service teachers, the difficulty has a further professional implication because they are expected not only to pass calculus but also to develop the confidence and conceptual understanding needed to teach advanced mathematics in future classrooms.

Readiness for calculus, therefore, involves more than performance on a single test of prior knowledge. It includes prerequisite knowledge, cognitive maturity for abstract reasoning, motivation, confidence and instructional support. From a constructivist perspective, learners build new mathematical ideas by reorganising prior knowledge through interaction, activity and feedback (Piaget, 1977; Vygotsky, 1978). Self-Determination Theory further suggests that competence, autonomy and relatedness influence learners' willingness to engage with cognitively demanding tasks (Deci and Ryan, 1985; Ryan and Deci, 2000). In this study, these theoretical perspectives were linked to the Markov modelling approach by treating readiness as an observable learning state that can change as students receive instruction, feedback and support. The Markov model then provides a way of representing those changes as transitions between readiness states rather than as a single static score.

The Ghanaian Colleges of Education context gives the problem particular relevance. Pre-service teachers enter college from diverse school backgrounds and with varying exposure to elective mathematics, concept-based instruction, tutoring, technology and institutional support. Weak readiness at the entry point may therefore affect not only immediate course performance but also the preparation of future mathematics teachers. Recent work continues to show that placement, readiness testing, prior mathematical preparation and institutional expectations influence calculus enrolment, progression and performance (Ryan et al., 2025; Stroumbakis and Robertson, 2025; Watson et al., 2025; Guerra et al., 2026). Similarly, recent studies of pre-service mathematics teachers emphasise the continuing gap between university-level mathematical work and the mathematical knowledge needed for school teaching (Menares-Espinoza et al., 2025; Wellberg, 2025).

Although existing readiness studies have classified students at entry or used prior achievement to predict later performance, fewer studies have followed the same students across successive readiness states during instruction. The gap is therefore both substantive and methodological. Substantively, there is limited evidence on whether pre-service teachers who begin calculus with weak preparation remain weak, improve, or regress during a semester. Methodologically, there is limited use of transition models that can show the probabilities of movement from one readiness state to another. Markovian modelling is well-suited to this purpose because it focuses on transitions between observed states and the probabilities associated with those transitions (Helske et al., 2024). The present study addresses this gap by modelling calculus readiness as a longitudinal transition process among pre-service teachers in Ghanaian Colleges of Education.

The study addressed three research questions: (1) What are the initial readiness states of students before learning calculus in Colleges of Education? (2) What are the probabilities of transitioning between readiness states over time? (3) What is the projected long-term readiness distribution among students in Colleges of Education under the observed transition dynamics?

Calculus readiness and prerequisite knowledge

Readiness to study calculus is closely related to students' mastery of prerequisite concepts. Carlson et al. (2015) argued that meaningful calculus learning requires coherent conceptions of functions, rate of change and covariation. Thompson and Harel (2021) similarly identified foundational ideas that shape students' subsequent engagement with calculus and explained how weaknesses in these ideas can constrain learning.

Empirical studies support this position. Sonnert et al. (2020) found that prior mathematics preparation predicted college calculus performance. Nortvedt and Siqveland (2019) reported that many beginning calculus and engineering students had weak basic mathematical knowledge, including competencies expected from earlier schooling. Peralta et al. (2020) also showed the usefulness of readiness instruments for identifying students' pre-calculus competence before difficulties become deeply embedded. More recently, Ryan et al. (2025) showed that mathematics placement information can shape Calculus 1 enrolment and persistence decisions, while Guerra et al. (2026) demonstrated the value and limitations of using compact mathematics-readiness indicators to guide early support in first-year quantitative courses.

Teaching, support and readiness development

Students' readiness can improve when teaching moves beyond procedural demonstration. Sencindiver (2020) emphasised the role of pre-calculus content knowledge and students' awareness of that knowledge in their success in calculus. Active-learning research further suggests that structured participation, peer discussion and problem-based activity can support students who enter with weaker backgrounds. Watson et al. (2025) found that active learning supported students with low pre-calculus proficiency, while Ng et al. (2020) showed that cooperative problem-based learning and peer assessment can improve undergraduate mathematics learning.

These findings align with constructivist learning theory, which positions learners as active participants in meaning-making. They also connect with motivational research indicating that competence beliefs and autonomous motivation can predict mathematics achievement (Wang et al., 2022). In calculus classrooms, students need opportunities to discuss, test, visualise and revise mathematical ideas, as well as feedback systems that allow them to view readiness as improvable.

Modelling readiness as a transition process

Many readiness studies classify students at a single time point or use readiness scores to predict later grades. Such approaches are useful but do not show how students move between levels of preparedness during instruction. A Markov chain model is appropriate when the focus is on movement through discrete states over time. In this study, readiness was classified into four states, and the probabilities of movement between adjacent time points were estimated.

Markov analysis can support education planning by distinguishing persistence, progress and regression. It can show whether instruction mainly raises average performance or whether it also changes the distribution of learners across readiness states. This evidence is useful for targeting bridging courses, tutorials, formative assessment, peer support and improvements in instructional quality. The present study, therefore, conceptualised calculus readiness as a transitional process.

MATERIALS AND METHODS

Research design and setting

The study used a quantitative longitudinal design to model changes in pre-service teachers' calculus readiness over one semester. It was conducted in five Colleges of Education in Ghana. The five colleges were purposively selected, with one college selected from each of the five zones in which the Colleges of Education had been grouped. This selection strategy was used to ensure zonal spread, include colleges offering the relevant mathematics programme, and secure complete longitudinal readiness data across the three measurement points. The purposive selection did not make the sample a simple random sample of all Colleges of Education in Ghana; rather, it strengthened contextual coverage by ensuring that each zone was represented.

Readiness was measured at three points: baseline before formal calculus instruction, mid-semester and end-of-semester. These time points enabled examination of the direction and stability of readiness movement during instruction, rather than relying on a single readiness classification.

Participants and sampling procedure

The accessible target population comprised 650 third-year pre-service teachers offering mathematics as an elective course in the five selected Colleges of Education. Based on Krejcie and Morgan's (1970) sample-size table, a sample of 242 would have been adequate for a population of 650. However, the study used 363 students to increase precision, strengthen representation across colleges, reduce the effect of unusable records, and improve the stability of transition estimates in a four-state Markov model. Because transition probabilities are estimated within rows of the transition matrix, a larger sample size was particularly useful for reducing sparse-cell problems and providing more reliable estimates for the smaller readiness categories.

Proportional allocation was used to determine the number of students selected from each college. The allocation followed the formula $n_h = (N_h/N)n$, where n_h is the sample for a college, N_h is the accessible population of that college, N is the total accessible population of 650, and n is the final sample of 363. After determining each college's allocation, simple random sampling was used to select the students from the college list. The unit of analysis was the individual student's readiness classification at each of the three measurement points. Only students with usable readiness classifications across all three time points were included in the Markov analysis.

Instrument and readiness classification

Students' readiness was assessed using three aligned assessment points. At baseline, students took the Thompson Rivers University Pre-Calculus Diagnostic Test. This test was selected because it assesses prerequisite domains relevant to calculus learning, including algebra, functions, graphs, trigonometry and mathematical reasoning. These domains align with mathematical competencies expected in the Ghanaian Senior High School and College of Education mathematics curriculum, making the test suitable for assessing students' entry readiness.

Mid-semester and end-of-semester assessments were conducted by the affiliate university, the University of Cape Coast, through the mid-semester quiz and end-of-semester examination. The comparison across the three assessment points was conceptual rather than item-by-item. The instruments were not identical, but all three assessed mathematical competencies related to calculus readiness and produced scores that could be converted to a common thirty-point scale.

To support comparability and replicability, each score was converted to a common thirty-point scale using the formula $S_{30} = (S_{raw}/S_{max})$, where S_{raw} is the student's raw score, and S_{max} is the maximum obtainable score for the assessment. Thus, the baseline diagnostic score was increased from 25 to 30 points, the mid-semester quiz from 20 to 30 points, and the end-of-semester examination from 60 to 30 points.

After conversion, the scores were grouped into four readiness levels: Unprepared (0-11), Partially Prepared (12-18), Prepared (19-25) and Highly Prepared (26-30). These bands were defined prior to analysis to reflect increasing mastery on a 30-point scale. Unprepared represented severe gaps in prerequisite knowledge;

Partially Prepared represented partial mastery with important gaps; Prepared represented adequate preparation for calculus learning; and Highly Prepared represented strong mastery with prospects for advanced engagement. The four bands also kept the state space small enough for stable Markov estimation while preserving educationally meaningful distinctions among students.

Validity, reliability and comparability considerations

The validity of the readiness classification was supported in three ways. First, the baseline diagnostic test and the University of Cape Coast assessments covered mathematics domains relevant to calculus learning. Second, the conversion to the thirty-point scale was linear and therefore preserved relative performance within each assessment. Third, the readiness bands were applied consistently across all three measurement points. Reliability was supported by the use of scored assessments administered in accordance with institutional procedures. Nevertheless, the study did not treat the three instruments as psychometrically identical. The results were interpreted as changes in readiness classification across aligned assessments, not as repeated administrations of the same instrument.

Analytical framework

Conceptually, calculus readiness was treated as an emergent learning state influenced by prior mathematical knowledge, cognitive development, teaching methods, self-efficacy and institutional support. Figure 1 summarises the relationship among these antecedent conditions, initial readiness, the transition through readiness states, and the projected steady-state distribution.

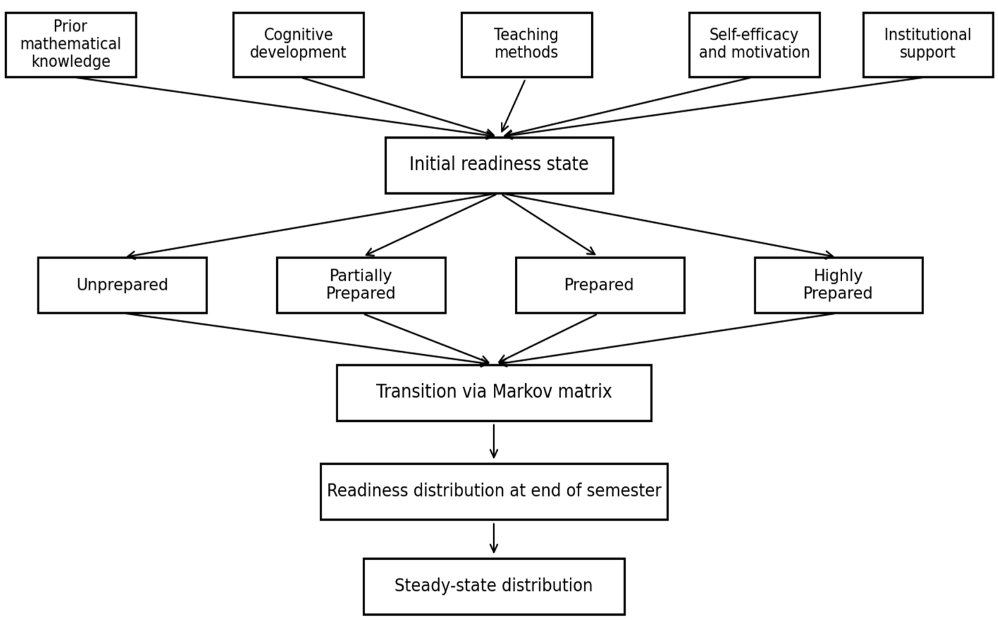

Figure 1. Conceptual framework for modelling calculus readiness transitions. Source: Field data (2025).

Analytically, readiness was modelled as a four-state discrete-time Markov chain. The four states were Unprepared, Partially Prepared, Prepared and Highly Prepared. Let X_t denote a student's readiness state at time t, where t = 0, 1 and 2 represent baseline, mid-semester and end-of-semester, respectively.

The probability that a student moves from readiness state i at time t to readiness state j at time t + 1 is $p_{ij} = \Pr(X_{t+1}=j \mid X_t=i)$. The full transition structure is represented by the matrix $P = [p_{ij}]$. If π_t denotes the row vector of students' proportions across the four readiness states at time t, then the distribution at the next time point is given by $\pi_{t+1} = \pi_t P$.

The long-run or steady-state distribution, denoted by π^*, satisfies $\pi^* = \pi^* P$ and the elements of π^* sum to one. In this study, the steady-state distribution was interpreted as a projected long-term readiness pattern under the observed transition probabilities, not as a direct causal prediction.

Markov model assumptions

The analysis was based on four assumptions. First, the readiness states were discrete and mutually exclusive. Second, the model was first-order, meaning that the next readiness state was assumed to depend primarily on the current state rather than the full prior history. Third, each row of the transition matrix was row-stochastic, so the transition probabilities from a given state summed to one. Fourth, the pooled steady-state analysis used a time-homogeneous summary matrix. Because the two observed transition periods occurred at different stages of the semester, the homogeneity assumption was treated as a modelling simplification. A chi-square homogeneity test was therefore conducted, and the steady-state results were interpreted with caution.

Data analysis

The first research question was answered using frequencies and percentages to describe baseline readiness. The second research question was answered by computing transition count matrices and row-normalised transition probability matrices for the baseline-to-mid-semester transition, the mid-semester-to-end-of-semester transition, and the overall baseline-to-end-of-semester movement.

For each transition period, the transition probability in row i and column j was estimated as $p_{ij} = n_{ij}/n_i$, where n_{ij} is the number of students who moved from state i to state j, and n_i is the number of students who began the transition period in state i. Approximate 95% confidence intervals were computed for selected transition probabilities to show the uncertainty around key estimates. A row-wise chi-square homogeneity test was also used to compare the baseline-to-mid-semester and mid-semester-to-end-of-semester transition patterns for rows with sufficient observations.

To answer the third research question, the one-step transitions were pooled across the two adjacent periods to estimate a time-homogeneous transition matrix. The pooled matrix was used as a descriptive summary of the observed readiness process. The steady-state distribution was obtained by solving $\pi^* = \pi^*P$ with the elements of π^* summing to one.

RESULTS

Initial readiness states before calculus instruction

The first research question examined students' readiness before they began learning calculus. Table 1 shows that baseline readiness was low, with most students classified below the desired readiness level. The absence of students in the Highly Prepared category and the small Prepared group indicates that formal calculus instruction began in a context of weak prerequisite preparation.

This distribution is consistent with studies showing that weaknesses in algebra, functions and pre-calculus concepts hinder calculus learning (Carlson et al., 2015; Thompson and Harel, 2021). Figure 2 provides a visual summary of the baseline readiness distribution.

Table 1. Initial readiness distribution at baseline (N = 363).

Readiness state	Frequency	Percentage
Unprepared	279	76.86
Partially prepared	66	18.18
Prepared	18	4.96
Highly prepared	0	0.00

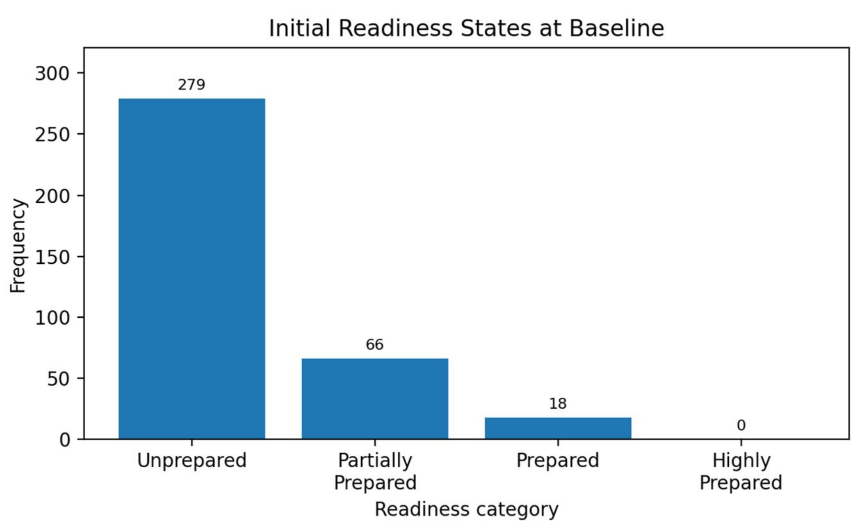

Figure 2. Initial readiness states at baseline. Source: Field data (2025).

Transition probabilities between readiness states

The second research question examined movement across readiness states. Count matrices are presented before probability matrices to show the number of students contributing to each estimated transition. The baseline-to-mid-semester transition showed substantial upward movement from the lower readiness states, particularly from Unprepared to Partially Prepared and from Partially Prepared to Prepared. The corresponding probability matrix in Table 3 shows the likelihood of each transition after row normalisation.

Table 2. Transition count matrix from baseline to mid-semester.

From/To	Unprepared	Partially prepared	Prepared	Highly prepared
Unprepared	117	121	38	3
Partially prepared	5	11	29	21
Prepared	1	0	7	10
Highly prepared	0	0	0	0

Table 3. Transition probability matrix from baseline to mid-semester.

From/To	Unprepared	Partially prepared	Prepared	Highly prepared
Unprepared	0.419	0.434	0.136	0.011
Partially prepared	0.076	0.167	0.439	0.318
Prepared	0.056	0.000	0.389	0.556
Highly prepared	-	-	-	-

The mid-semester-to-end-of-semester transition was more variable. Table 4 shows that some students improved, while others regressed or remained the same. This pattern was especially evident among students in the lower and middle readiness categories. Table 5 indicates that the Prepared and Highly Prepared categories were more stable than the lower categories, although regression was still observed.

Table 4. Transition count matrix from mid-semester to end-of-semester.

From/To	Unprepared	Partially prepared	Prepared	Highly prepared
Unprepared	58	38	26	1
Partially prepared	40	50	40	2
Prepared	3	13	45	13
Highly prepared	1	2	12	19

Table 5. Transition probability matrix from mid-semester to end-of-semester.

From/To	Unprepared	Partially prepared	Prepared	Highly prepared
Unprepared	0.472	0.309	0.211	0.008
Partially prepared	0.303	0.379	0.303	0.015
Prepared	0.041	0.176	0.608	0.176
Highly prepared	0.029	0.059	0.353	0.559

Table 6 reports selected uncertainty estimates for the transition probabilities. The confidence intervals show that estimates based on larger starting rows, such as the Unprepared baseline row, were relatively more precise than estimates based on smaller rows, such as the Prepared baseline row. This is important because only 18 students were in the Prepared state at baseline, and none were in the Highly Prepared state.

Table 6. Selected transition probabilities with approximate 95% confidence intervals.

Transition period	Movement	Probability	Approximate 95% CI
Baseline-to-mid	Unprepared -> Partially Prepared	0.434	0.377 - 0.492
Baseline-to-mid	Unprepared -> Prepared	0.136	0.101 - 0.181
Baseline-to-mid	Partially Prepared -> Prepared	0.439	0.326 - 0.559
Baseline-to-mid	Prepared -> Highly Prepared	0.556	0.337 - 0.754
Mid-to-end	Unprepared -> Unprepared	0.472	0.386 - 0.559
Mid-to-end	Partially Prepared -> Partially Prepared	0.379	0.301 - 0.464
Mid-to-end	Prepared -> Prepared	0.608	0.494 - 0.711
Mid-to-end	Highly Prepared -> Highly Prepared	0.559	0.395 - 0.711

The overall baseline-to-end-of-semester matrices summarise the net movement across the full semester. Table 7 shows that initial readiness was associated with final readiness, but it did not fully determine final status. Table 8 shows that students who began in higher readiness states had stronger probabilities of ending in the Prepared or Highly Prepared categories.

Overall, the transition results show that readiness was fluid. Students could progress, remain at the same level or regress depending on their starting point and the instructional period. The stronger upward movement in the early part of the semester suggests that early instructional support helped move some students out of the least-developed readiness category, while the later variability may reflect the increasing abstraction of calculus content.

Table 7. Combined transition count matrix from baseline to end-of-semester.

From/To	Unprepared	Partially prepared	Prepared	Highly prepared
Unprepared	99	94	84	2
Partially prepared	2	9	37	18
Prepared	1	0	2	15
Highly prepared	0	0	0	0

Table 8. Combined transition probability matrix from baseline to end-of-semester.

From/To	Unprepared	Partially prepared	Prepared	Highly prepared
Unprepared	0.355	0.337	0.301	0.007
Partially prepared	0.030	0.136	0.561	0.273
Prepared	0.056	0.000	0.111	0.833
Highly prepared	-	-	-	-

Homogeneity check and pooled transition matrix

Because the baseline-to-mid-semester and mid-semester-to-end-of-semester matrices showed visibly different patterns, a row-wise chi-square homogeneity check was conducted before interpreting the pooled transition matrix. The Unprepared row did not differ significantly across the two periods, chi-square(3) = 6.98, p = 0.072. However, the Partially Prepared row differed significantly, chi-square(3) = 53.56, p < 0.001, and the Prepared row also differed significantly, chi-square(3) = 12.83, p = 0.005. Combining the estimable rows also indicated non-homogeneity across the two periods, chi-square(9) = 73.36, p < 0.001.

This result means that the pooled matrix should not be read as evidence that the two transition processes were identical. It was retained only as a descriptive, one-step summary for projecting a cautious, steady-state distribution based on the average observed dynamics. The pooled one-step transition probability matrix is presented in Table 9.

Table 9. Pooled one-step transition probability matrix.

From/To	Unprepared	Partially prepared	Prepared	Highly prepared
Unprepared	0.4353	0.3955	0.1592	0.0100
Partially prepared	0.2273	0.3081	0.3485	0.1162
Prepared	0.0435	0.1413	0.5652	0.2500
Highly prepared	0.0294	0.0588	0.3529	0.5588

Projected long-term steady-state distribution

The third research question examined the projected long-term readiness distribution. Solving for the steady-state distribution from the pooled matrix produced the proportions reported in Table 10. Under the average observed transition dynamics, the readiness process would eventually place most students in the Prepared or Highly Prepared categories, although a meaningful minority would remain below the desired readiness level.

Table 11 compares the observed end-of-semester distribution with the steady-state distribution. The comparison suggests that the process had not reached equilibrium by the end of the semester. Figure 3 shows this contrast visually. Because the pooled matrix is based on only two observed transition periods and because the homogeneity check showed non-identical transition patterns, the steady-state distribution should be understood as an educational planning projection rather than a guaranteed future outcome.

Table 10. Steady-state readiness distribution.

Readiness state	Steady-state proportion (Percentage)
Unprepared	0.1185 (11.8)
Partially prepared	0.1774 (17.7)
Prepared	0.4179 (41.8)
Highly prepared	0.2862 (28.6)

Table 11. Observed end-of-semester distribution compared with the steady-state distribution.

Readiness state	Observed at the end of the semester (Percentage)	Steady state (Percentage)
Unprepared	0.281 (28.1)	0.118 (11.8)
Partially prepared	0.284 (28.4)	0.177 (17.7)
Prepared	0.339 (33.9)	0.418 (41.8)
Highly prepared	0.096 (9.6)	0.286 (28.6)

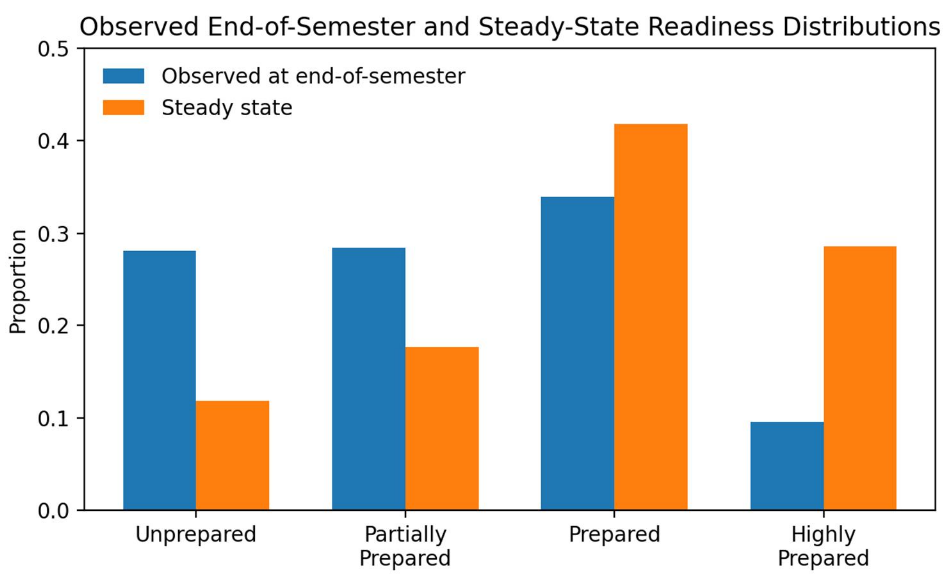

Figure 3. Observed end-of-semester and steady-state readiness distributions. Source: Field data (2025).

DISCUSSION

The findings support earlier studies showing that prerequisite knowledge in algebra, functions, trigonometry, rate of change and covariation is central to calculus learning (Carlson et al., 2015; Thompson and Harel, 2021; Sonnert et al., 2020). The baseline distribution in this study indicates that many pre-service teachers began calculus with weak preparation. In the Ghanaian Colleges of Education context, such low readiness should not be interpreted only as an individual problem. It also reflects the transition from prior schooling to the expectations of mathematics teacher education.

The use of the Thompson Rivers University diagnostic test served the same broad purpose as other readiness instruments, namely identifying pre-calculus competence before students encounter sustained difficulty (Peralta et al., 2020). Its emphasis on algebra, functions, graphs, and trigonometry made it an appropriate baseline measure, as these domains are relevant to the Ghanaian mathematics curriculum. However, the study used three different assessment sources. The results should therefore be interpreted as transitions across aligned readiness classifications rather than changes on repeated administrations of the same test.

The strong early upward movement is consistent with studies suggesting that structured teaching support can benefit students who begin with low readiness (Ng et al., 2020; Watson et al., 2025). It is also consistent with constructivist theory, which views learning as the reorganisation of prior knowledge through activity, interaction and feedback (Piaget, 1977; Vygotsky, 1978). The latter transition matrix, however, shows that early gains do not always persist. As students encounter more abstract calculus ideas such as limits, derivatives and integrals, some may regress or remain in lower readiness categories.

Regression during the second half of the semester may have occurred because later calculus tasks required greater conceptual coordination, symbolic fluency and independent strategy selection than the early instructional tasks. Students who improved during the first half of the semester may still have had a fragile understanding of functions, algebraic manipulation or trigonometric reasoning. Once the course content became more abstract, those weaknesses could reappear. Assessment conditions, differences in tutorial support, study habits and cognitive load may also have contributed to the observed regression.

This pattern connects with Sencindiver’s (2020) emphasis on both pre-calculus knowledge and metacognitive awareness. Students may show initial improvement when tasks are familiar or closely supported, but later struggle when they must select strategies, monitor understanding and adapt to new levels of abstraction. Upward transitions may therefore require more than content exposure; they may also require feedback, self-monitoring and sustained opportunities to consolidate conceptual understanding.

The results can be interpreted cautiously through the perspectives of Self-Determination Theory and self-regulated learning, as competence beliefs, motivation, and strategic effort may help explain why some students progress while others regress (Ryan and Deci, 2000; Schunk and Greene, 2018; Wang et al., 2022). Nevertheless, the present study did not directly measure motivation, autonomy, relatedness or self-regulated learning strategies. Future studies should combine transition modelling with questionnaire and interview data to examine how these factors relate to readiness movement.

The Markov chain approach adds value by moving beyond averages and static classification. Transition matrices make it possible to observe persistence, progress and regression across readiness states. They also help identify readiness groups that are unlikely to progress without additional support. The model is nevertheless a simplification. The assumption that the next state depends primarily on the current state does not fully capture the possible influence of attendance, teaching quality, assessment format, peer support, study habits and prior educational history.

The small number of students in the Prepared category at baseline and the absence of students in the Highly Prepared category also have implications for interpretation. Transition probabilities from small rows are more uncertain, as shown by the wider confidence interval for the first transition from Prepared to Highly Prepared. The absence of Highly Prepared students at baseline means that the first-period transition pattern for that state could not be estimated directly. These limitations justify a cautious reading of the higher-readiness transition estimates.

The steady-state results should therefore be read as projections under the average observed transition dynamics, not as guarantees of future outcomes. The projected distribution indicates that a larger share of students could stabilise in the Prepared and Highly Prepared categories if similar dynamics continued. At the same time, a substantial minority would remain below the desired readiness level. Colleges of Education should therefore sustain support throughout the semester rather than treating early improvement as sufficient evidence of readiness.

The educational significance of the steady-state distribution lies in its use in planning. If approximately one-third of students remain projected below the desired readiness level, then policy and curriculum responses should not end with a single diagnostic test. Colleges of Education could use transition information to design early warning systems, allocate tutorials proportionally to need, strengthen pre-calculus bridging, and monitor whether interventions change the transition probabilities over time. At the curriculum level, the findings point to the need for stronger alignment between Senior High School mathematics preparation, College of Education mathematics courses, and the expectations of calculus instruction.

Conclusion

This study applied a Markov chain model to examine pre-service teachers' readiness to learn calculus concepts in Ghanaian Colleges of Education. The baseline results showed that many students entered the course with weak prerequisite preparation. The transition matrices showed that readiness changed over the semester, with strong early improvement but greater variability later. These findings show that low readiness is not necessarily permanent, but improvement must be consolidated through sustained instructional support.

The study contributes a transition-based view of calculus readiness. For pre-service teachers, readiness matters not only for examination performance but also for future teaching competence. Colleges of Education should therefore combine diagnostic testing, bridging activities, conceptual instruction, peer learning and formative feedback throughout calculus instruction.

RECOMMENDATIONS

Colleges of Education should conduct calculus-readiness diagnostics before formal instruction begins, using instruments that assess algebra, functions, graphing, trigonometry and mathematical reasoning. Students classified as Unprepared or Partially Prepared should receive structured pre-calculus support before and during the calculus course. Tutors should emphasise conceptual understanding as well as procedural fluency by using visualisation, real-world applications, group work and formative feedback. Support should continue beyond the beginning of the semester because the transition results show that early gains can weaken as course abstraction increases.

Teacher education programmes should also use transition evidence to monitor the effectiveness of support. If an intervention is working, the transition matrices should show fewer downward movements, stronger persistence in the Prepared and Highly Prepared states, and reduced persistence in the Unprepared state. Programme leaders can therefore use repeated transition analysis as part of quality assurance for calculus teaching.

LIMITATIONS

The study was limited to five purposively selected Colleges of Education in Ghana. Although selecting one college from each zone improved contextual coverage, the college selection was not a national random sample. The findings should therefore be interpreted as evidence from the selected zonal colleges rather than as a direct estimate for all Colleges of Education.

The study also used test results from three assessment points. Although the Thompson Rivers University diagnostic test and the University of Cape Coast mid-semester and end-of-semester assessments were aligned to the general construct of calculus readiness and converted to a common thirty-point scale, they were not identical instruments. The transition results should therefore be interpreted as changes in readiness classification across aligned assessment points rather than as evidence from repeated administration of the same test.

The Markov model simplified a complex learning process. The first-order assumption did not include each student's full learning history, and the pooled steady-state projection relied on an average transition matrix derived from only two observed transition periods. The homogeneity check showed that some transition rows differed across the two periods, so the steady-state findings should be used for planning and reflection rather than direct prediction.

The study did not include qualitative classroom evidence or questionnaire-based predictors of readiness transitions. Future studies should link diagnostic scores, questionnaire responses, and demographic variables using common identifiers to model individual-level predictors of movement more precisely. Further research should also test intervention scenarios empirically by comparing transition matrices before and after bridging courses, peer-assisted learning or technology-supported instruction.

Ethics statement

Ethical clearance for the study was obtained from the Humanities and Social Sciences Research Ethics Committee (HuSSREC) of Kwame Nkrumah University of Science and Technology with reference number HUSSREC/AP/232/VOL.4. All participants provided written informed consent before inclusion in the study. Consent was also obtained from department heads, lecturers and participating students in accordance with the committee's ethical standards.

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