Effect of trajectory planning duration time on correct dynamic response selection of micro-robot for surgical application

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ABSTRACT

The applications of robotics in medical field have increased extensively in the last two decades. The aim of this paper is to study the effect of trajectory planning method duration time on correct dynamic response selection of six degrees of freedom micro-robot intended for surgery applications using two different methods of trajectory planning with different four duration times (5, 10, 20 and 60 s). The kinematic equations of motion were obtained using Denavit-Hartenberg representation. The dynamic equations of motion, which are important for the proper design of robot controller, were first derived using the Lagrangian-Euler technique. Then, the required hub torque to move each joint was calculated for the motor selection. The trajectory planning was derived using two different methods of trajectory planning with different four duration times (5, 10, 20 and 60 s). These methods are the fifth-order polynomial and soft motion trajectory planning. A comparison of the trajectory planning and dynamic response for the different four duration times was carried out of the two different methods of trajectory planning to choose the best duration time and the best method of trajectory planning that gives the correct dynamic response, smooth set trajectory planning and best performance of the robot under investigation. The simulation results were obtained using MATLAB software.

Keywords: Trajectory planning, dynamic response, micro-robot, surgery applications, Lagrangian-Euler technique, fifth-order polynomial, soft motion, duration time, correct dynamic response selection, MATLAB software.

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INTRODUCTION

Robots are widely used in medical field for minimally invasive surgery efficiently and accurately. Minimally invasive surgery is an innovative approach that allows reducing patient trauma, postoperative pain and recovery time (Cepolina, 2005). The kinematics and dynamics analysis of the robot are very important for any applications. The direct aim is to properly select the workspace and the actuator size. Konietschke et al. (2006) designed a minimally invasive robot for heart surgery. They concentrated mainly in the kinematics analysis and the workspace of the robot. Tsai and Hsu (2004) investigated a parallel surgical robot having six degree-of-freedom (DOF). They performed the kinematic analysis only to obtain the workspace of surgical robot and control it by Fuzzy Logic method. Standard workspaces of minimally invasive as well as open surgical procedures are considered and optimization criteria are derived. Miller and Christensen (2003) focused on the dynamic of the Multi-rigid-body robot using Newton’s method and applied the results to design the controller. Featherstone and Orin (2000) investigated the robot dynamics equations using Newton-Euler algorithm to obtain the equations of motion for the robot arm.

KINEMATIC ANALYSIS

The geometrical model of the surgical robot in this study has 6 degrees of freedom (DOF) and an extra one for tool action. The end-effector has 3 rotations (Roll, Pitch and Yaw) as shown in Figure 1a and the frame assignment...
of 6-DOF surgical manipulator is represented in Figure 1b based on Denavit-Hartenberg.

The 6-DOF manipulator kinematic parameters derived are shown in Table 1. The flowchart representing the sequence of generating the link transformations matrix is shown in Figure 2.

The homogeneous transformation matrices can be given as:

![Figure 1. Geometric model of surgical manipulator.](image)

<table>
<thead>
<tr>
<th>Link #</th>
<th>( \theta_i )</th>
<th>( d_i )</th>
<th>( a_i )</th>
<th>( \alpha_i )</th>
<th>( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \theta_1 )</td>
<td>0</td>
<td>( L_1 )</td>
<td>0</td>
<td>( ^0T_1 )</td>
</tr>
<tr>
<td>2</td>
<td>( \theta_2 )</td>
<td>0</td>
<td>( L_2 )</td>
<td>90</td>
<td>( ^1T_2 )</td>
</tr>
<tr>
<td>3</td>
<td>( \theta_3 )</td>
<td>0</td>
<td>0</td>
<td>-90</td>
<td>( ^2T_3 )</td>
</tr>
<tr>
<td>4</td>
<td>( \theta_4 )</td>
<td>( L_3 )</td>
<td>0</td>
<td>90</td>
<td>( ^3T_4 )</td>
</tr>
<tr>
<td>5</td>
<td>( \theta_5 )</td>
<td>0</td>
<td>0</td>
<td>-90</td>
<td>( ^4T_5 )</td>
</tr>
<tr>
<td>6</td>
<td>( \theta_6 )</td>
<td>( L_4 )</td>
<td>0</td>
<td>0</td>
<td>( ^5T_6 )</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
^0T_1 & = \begin{bmatrix}
C_1 & -S_1 & L_1C_1 & 0 \\
S_1 & C_1 & L_1S_1 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \\
^3T_4 & = \begin{bmatrix}
C_4 & -S_4 & 0 & 0 \\
S_4 & C_4 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \\
^5T_6 & = \begin{bmatrix}
C_6 & -S_6 & 0 & 0 \\
S_6 & C_6 & 0 & 0 \\
0 & 0 & 1 & L_4 \\
0 & 0 & 0 & 1
\end{bmatrix}
\end{align*}
\]

\[P_x = \begin{bmatrix}
\begin{array}{ccccc}
(C_1 S_2 + S_1 C_2) & C_1 & S_1 & -S_1 & S_3 \times S_1 \\
(-C_1 S_2 + S_1 C_2) & -S_3 & a_3 & a_3 & L_3 \\
S_1 & 0 & 0 & 0 & 1
\end{array}
\end{bmatrix}
\]

\[P_y = \begin{bmatrix}
\begin{array}{ccccc}
(C_1 S_2 + S_1 C_2) & C_1 & S_1 & S_1 & S_3 \times S_1 \\
(-C_1 S_2 + S_1 C_2) & -S_3 & 0 & 0 & L_3 \\
S_1 & 0 & 0 & 0 & 1
\end{array}
\end{bmatrix}
\]

\[P_z = \begin{bmatrix}
\begin{array}{ccccc}
-S_1 & C_1 & S_1 & S_1 & S_3 \times S_1 \\
S_1 & 0 & 0 & 0 & L_3 \\
C_1 & 0 & 0 & 0 & 1
\end{array}
\end{bmatrix}
\]

Where: \( C_i = \cos \theta_i \), \( S_i = \sin \theta_i \)

**THE TRAJECTORY PLANNING**

Trajectory planning aims at driving the joint from an initial
The most common techniques for trajectory planning for industrial robots are polynomial of different orders, Cubic and B-splines, linear segments with parabolic blends and the soft motion trajectory (Ata, 2007). In this paper, two different methods are applied here to design the joints trajectories fifth order polynomial and soft motion trajectory. These two trajectories have the same initial and final angles and different four duration times (5, 10, 20 and 60 s) are applied to choose the best duration time given the correct dynamic response, but they differ in the acceleration and the jerk. After designing the joints trajectories, the hub torques of the robot actuators can be simulated using MATLAB. For simulation analysis, the parameters of six joints are shown in Table 2. The flowchart representing the sequence of generating the trajectory planning is shown in Figure 3.

### Table 2. Robot parameters for trajectory planning.

<table>
<thead>
<tr>
<th>Link#</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_i$ (deg)</td>
<td>-180°</td>
<td>-90°</td>
<td>0</td>
<td>-180°</td>
<td>-90°</td>
<td>-180°</td>
</tr>
<tr>
<td>$\theta_f$ (deg)</td>
<td>180°</td>
<td>90°</td>
<td>180°</td>
<td>180°</td>
<td>90°</td>
<td>180°</td>
</tr>
<tr>
<td>L (mm)</td>
<td>0</td>
<td>50</td>
<td>15</td>
<td>36</td>
<td>0</td>
<td>39.5</td>
</tr>
<tr>
<td>$\dot{\theta}_i$ (deg/s²)</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$\ddot{\theta}_i$ (deg/s³)</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
</tr>
<tr>
<td>$\dot{\theta}_f$ (deg/s)</td>
<td>135</td>
<td>70</td>
<td>70</td>
<td>135</td>
<td>70</td>
<td>135</td>
</tr>
<tr>
<td>$\ddot{\theta}_f$ (deg/s³)</td>
<td>80</td>
<td>40</td>
<td>40</td>
<td>80</td>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td>$\dot{\theta}_{max}$ (deg/s)</td>
<td>160</td>
<td>80</td>
<td>80</td>
<td>160</td>
<td>80</td>
<td>160</td>
</tr>
<tr>
<td>$\dot{\theta}_{max}$ (deg/s³)</td>
<td>80</td>
<td>40</td>
<td>40</td>
<td>80</td>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td>$t_i$ (s)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$t_f$ (s)</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

### Fifth-order polynomial trajectory planning

Increasing the order of the polynomial allows the designer to control the initial and final acceleration as well as the jerk. The third order trajectory only takes into account starting and end velocities. The equations of the fifth order polynomial takes into account starting and end accelerations. In this case, the total number of required boundary conditions is six. The initial and final velocities are assumed to be zero (Rest-to-Rest maneuvering) and the trajectory is given by Herrera and Sidobre, (2006).

### Governing equations

$$\theta(t) = C_0 + C_1t + C_2t^2 + C_3t^3 + C_4t^4 + C_5t^5$$  \(1\)

Where $C_0, C_1, C_2, C_3, C_4$ and $C_5$ are coefficients to be calculated.
determined from the initial and final conditions as follows:

\[
\begin{align*}
C_0 &= \theta_i, \quad C_1 = \dot{\theta}_i, \quad C_2 = \ddot{\theta}_i, \\
C_3 &= \frac{200\theta_i - 200\dot{\theta}_i - (8\ddot{\theta}_i + 12\dddot{\theta}_i) t_f - (3\dddot{\theta}_i - \dddot{\theta}_i) t_f^2}{2t_f^3} \\
C_4 &= \frac{300\theta_i - 300\dot{\theta}_i + (14\ddot{\theta}_i + 16\dddot{\theta}_i) t_f + (3\dddot{\theta}_i - 2\dddot{\theta}_i) t_f^2}{2t_f^4} \\
C_5 &= \frac{12\theta_i - 12\dot{\theta}_i - (6\ddot{\theta}_i + 6\dddot{\theta}_i) t_f + (\dddot{\theta}_i - \dddot{\theta}_i) t_f^2}{2t_f^5}
\end{align*}
\]

Where: \( \theta_i \) and \( \theta_f \) are the initial and final values of each joint angle and \( t_f \) is the time duration.

**Soft motion trajectory planning**

**Governing equations**

The soft motion trajectory introduced by Herrera and Sidobre (2006) for service robots consists of seven segments to produce soft motion and guarantee the end-effector's motion characteristics (jerk, acceleration, velocity and position). The soft trajectory in the general case is given by:

**The motion with a maximum jerk \((J_{\text{max}})\):**

\[
\begin{align*}
\theta(t) &= \theta_0 + \dot{\theta}_0 t + \frac{1}{2} \ddot{\theta}_0 t^2 + \frac{1}{6} J_{\text{max}} t^3 \\
\dot{\theta}(t) &= \dot{\theta}_0 + \ddot{\theta}_0 t + \frac{1}{2} \dddot{\theta}_0 t^2, \quad \ddot{\theta}(t) = \dddot{\theta}_0 + J_{\text{max}}, \quad J(t) = J_{\text{max}}
\end{align*}
\]

**The motion with a maximum acceleration \((A_{\text{max}})\):**

\[
\begin{align*}
\theta(t) &= \theta_0 + \dot{\theta}_0 t + \frac{1}{2} \ddot{\theta}_0 t^2 + \frac{1}{2} \dddot{\theta}_0 t^2 \\
\dot{\theta}(t) &= \dot{\theta}_0 + \ddot{\theta}_0 t + \frac{1}{2} J_{\text{max}} t^2, \quad \ddot{\theta}(t) = \dddot{\theta}_0 + J_{\text{max}}, \quad J(t) = 0
\end{align*}
\]

Finally, the equations for the motion with a maximum velocity \((V_{\text{max}})\):

\[
\dot{\theta}(t) = \dot{\theta}_0 + \dddot{\theta}_0 t
\]

---

**Figure 3. Flowchart of the trajectory planning. Position 1: for Fifth-Order Polynomial trajectory planning. Position 2: for Soft motion trajectory planning.**
\[ \ddot{q}(t) = \ddot{q}_{\text{max}}, \quad \dot{q}(t) = 0, \quad q(t) = 0 \]

Where \( J(t), \dot{q}(t), \ddot{q}(t), \theta(t) \) represents angular jerk, acceleration, velocity, and position respectively and \( \theta_0, \dot{\theta}_0, \ddot{\theta}_0 \) are the initial conditions.

**DYNAMIC ANALYSIS**

Manipulator dynamics is concerned with the equations of motion, the way in which the manipulator moves in response to torques applied by the actuators, or external forces. The history and mathematics of the dynamics of serial-link manipulators are well covered in the literature. The equations of motion for an n-axis manipulator are given by Niku (2001):

\[ Q = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) \]  \hfill (5)

Where:
- \( q \) is the vector of generalized joint coordinates.
- \( \dot{q} \) is the vector of joint velocities.
- \( \ddot{q} \) is the vector of joint accelerations.
- \( M \) is the inertia matrix.
- \( C \) is the Coriolis and Centrifugal matrix.
- \( G \) is the gravity matrix.
- \( Q \) is the vector of generalized force associated.

The Lagrang–Euler is utilized here to calculate the kinetic energy, potential energy and derive the dynamic equations in symbolic form using MATLAB symbolic Toolbox for the six-degree-of-freedom robot because the multiple-degree-of-freedom robot has equations very long and complicated. The equations of motion can be given in a compact form as in Niku (2001):

\[ T_i = \sum_{j=1}^{n} D_{ij} \ddot{q}_j + I_i(q_{\text{act}}) \ddot{q}_i + \sum_{j=1}^{n} \sum_{k=1}^{n} D_{ijk} \dot{q}_j \dot{q}_k + D_i \]  \hfill (6)

\[ D_{ij} = \sum_{p=\text{max}(i,j)}^{n} \text{Trac}(U_{pj}U_{pi}^{T}) , \]

\[ D_{ijk} = \sum_{p=\text{max}(i,j,k)}^{n} \text{Trac}(U_{pjk}U_{pi}^{T}) , \]

\[ D_i = \sum_{p=1}^{n} -m_p \bar{g}^T U_{pj} \bar{r}_p , \]

where:
- \( J \) is the moment of inertia matrix for each link.
- \( n \) is the total number of links.
- \( i \) is the link number.
- \( j \) is the total number of joints.
- \( k \) is the coefficient \( (1, 2, 3... n) \), \( p \) is the coefficient \( (1, 2, 3... n) \).

For the MATLAB simulation, the parameters of the robotic arm are given in Table 3.
DYNAMIC RESPONSE SELECTION

Fifth-order polynomial trajectory planning

Figures 4 and 5 show comparisons between the orientation, velocity, acceleration, and torque for different four duration times (5, 10, 20 and 60 s) of the first joint of the robot. As clearly shown in Figure 3, the orientation behavior increases gradually with time from the initial angle to the final angle for the times (5, 10 and 20 s) only. But for the time 60 s, the orientation increases gradually from the initial angle to 40° then decreases to -40° and then increases reaching the final angle. And, if the time increases the velocity required decreases. This is similar as to what is shown in Figure 5 which presents the original torque history that has considerable fluctuations. It is clear that the highest hub torque is for joint 1 while actuator torque of joint 6 is the lowest.

From Figures 4 and 5, the optimum final duration time required to complete the process for the robot can be selected. It is seen from the figures that the trajectory planning of the robot for the times (5, 10 and 20 s) has the same properties, that is, the velocity is inversely proportional to time. The orientation behavior increased gradually with the time from the initial angle to the final angle. Where the time 60 s is omitted from selected. From Figure 5, for the torque history in the time (5, 10, 20 and 60 s), it is found for $t_f = 5\, s$, the dominant part in the torque history is the inertia matrix. Increasing the final time to 10, 20 and 60 s shifts the dominant term from inertia matrix to Centrifugal and Coriolis matrices. This is
due to the vanishing of the acceleration at most of the joint trajectory. Another consequence of increasing the final time is the dramatic change in the peak value of the joint torque which requires big actuator size for the same task. The time 5 s to complete the results of the surgical robot can be selected, since this time's orientation behavior was increased gradually with the time from the initial angle to the final angle and the torque history curves were affected by the inertia of the link of robot.

**Soft motion trajectory planning**

Figures 6 and 7 show comparisons between the orientation, velocity, acceleration, jerk, torque and four different duration times (5, 10, 20 and 60 s) of the first joint of the surgical robot. It is clearly shown in Figure 5 that the orientation behavior increases gradually with the time from the initial angle to the final angle and as the time increases the velocity required decreases and also the acceleration, that is, the inertia of the robot link decreases. It is also clearly shown in Figure 7 that the original torque history has considerable fluctuations. It is clear that the highest hub torque is for joint one while actuator torque of joint 6 is the lowest.

From Figures 6 and 7, we can select the optimum final time required to complete the process for the robot. By inspection of Figure 6, we find the trajectory planning three segments: maximum jerk, maximum acceleration and maximum velocity. The maximum acceleration and maximum velocity segments were very important segments because the velocity in the maximum velocity segment is constant and the acceleration was zero, that is, no inertia of the link of the robot in this segment of time and the acceleration in the maximum acceleration segment is constant and the jerk of the link of the robot was zero. It is therefore better to have a larger period for this segment. It is seen from the figures that the segments were released from time duration 60 s but the the maximum velocity segment only was released from time duration 10 s. Where the times (10 and 60 s) is omitted from those selected, that is, the times (10 and 60 s) have properties that were not satisfactory for trajectory of robot. And by inspection of Figure 7 for the torque history in the times 5 and 20 s, we find for $t_j = 5$ s, the dominant part in the torque history is the inertia matrix.
Increasing the final time duration to 20 s, shift the dominant term from inertia matrix to Centrifugal and Coriolis matrices since the effect of angular velocity will be obviously high. This is due to the vanishing of the acceleration at most of the joint trajectory. Another consequence of increasing the final time duration is the dramatic change in the peak value of the joint torque which requires big actuator size for the same task. So, we can select the time duration 5 s to complete the results of the robot because the torque history curve was affected by the inertia and it has the important segment, that is, maximum acceleration and maximum velocity segments.

CONCLUSIONS

As previously stated, the aim of this paper is to study the effect of trajectory planning method duration time on correct dynamic response selection of six degrees of freedom micro-robot intended for surgery applications using two different methods of trajectory planning with different four duration times (5, 10, 20 and 60 s). These methods were the fifth-order polynomial and soft motion trajectory planning. A comparison of the results was held to choose the best duration time and the best method of trajectory planning that gives the correct dynamic response, smooth set trajectory planning and best performance of the robot under investigation.

It was clearly shown in this work that the best method of trajectory planning that gives the smooth set trajectory planning and best performance of the robot under investigation, was the soft motion trajectory planning, because the most important reason for this selection was the torque history that has the lowest number ever of shootings, and the shooting was distributed regularly over the period of time unlike the other methods which have a long number of shootings and were distributed randomly. Also, the reason for selecting this method was disappearance of the shooting quite before the final time.
of trajectory, that is, the steady state time. The behavior resulting from this method was very important in design if the controller of this type of surgical robot.

REFERENCES