African Journal of Engineering Research
Vol. 2(1), pp. 8-13, February 2014
ISSN: 2354-2144
Full Length Research Paper

Analysis and prediction of green permeability values in sand moulds using multiple linear regression model

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Accepted 10 February, 2014

ABSTRACT

The analysis and prediction of green permeability values in sand moulds using multiple linear regression model has been undertaken. Empirically generated data in Sand Testing Laboratory, National Metallurgical Development Centre, Jos, was used for the study. The model described the relationship existing between the two variables of clay and moisture content and green permeability, the dependent variable. This was compared with existing theories on the relationship between green permeability and the two variables. A mathematical model was developed for the prediction of green permeability; it was tested and proved to be a good estimation tool for estimating green permeability values on the foundry shop floor.

Keywords: Green permeability, analysis, prediction, model, multiple linear regressions.

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INTRODUCTION

Green permeability is one of the properties of green sand moulds. Green permeability is controlled by a number of factors which include; sand particle size and shape, water content of the moulding mixture and clay content of the moulding mixture. This has been confirmed by several authors (Ihom, 2012; Khanna, 2008; Ihom et al., 2009). Jain (2009) outlines a number of factors upon which green sand permeability depends, and these are size of the grains, compactness density, moisture content in the moulding sand and bonding content (amount of binder or clay). He stressed that "the sand used for casting must be porous enough, so as to allow the gaseous material, water and steam vapours to escape freely when the molten metal is poured into the mould. Molten metal contains some dissolved gases which are evolved on solidification. Molten metal in contact with moist sand forms steam and vapour, which must find passage to escape completely. Insufficient porosity of moulding sand leads to casting defects such as gas holes and pores." The moulder has some control over permeability; hard ramming lowers the permeability, but this is relieved by liberal venting. The lowering of permeability as a result of ramming had earlier been investigated by some researchers (Brown, 1994; Heine et al., 1967), they revealed in their work that as the number of rams increase green permeability decreases.

Jain explained that green permeability decreases with decrease in grain size and increases with increase in grain size, the relationship is directly proportional. Permeability decreases with increase in clay content; the relationship is inversely proportional, however under the presence of ample moisture this relationship may not be drastic as when the moisture is low. The influence of moisture on permeability is such that as moisture is increased permeability also increases until an optimum point or peak is reached, where there after any increase in moisture results in decrease in permeability, this relationship was confirmed by Jain for both round and angular shaped sands (Jain, 2009). Other researchers (Zrimsek and Vingas, 1961; Chakoraborty and Dhindaw, 1979, 1985) who studied the permeability of green sand moulds using empirical methods and statistical approaches also agree with Jain (2009) and Ihom et al. (2011). Factors affecting permeability have individual

Table 1. Green permeability data with variation in clay and moisture content.

S/No	% Clay content variation X_1	% Moisture content variation X_2	Permeability Y_p (AFS perm unit)
1	1.5	2	310
2	2	3	280
3	3	4	300
4	4	5	320
5	5	6	310
6	6	7	300
7	7	8	270

influence as well as joint or combined influence on permeability; this was investigated by Ihom et al. (2006), the authors revealed that 'various sand systems are in use in the metal casting industry that have been investigated in the past. Such investigations were carried out mostly by classical methods of single factor experiments involving a large number of trials; yet, the interactions between the variable factors have not been clearly understood. The combined influence of moisture and clay on green permeability had a positive coefficient of determination r^2, coefficient of correlation r, and coefficient of multiple determination R^2. This result agrees with studies on the effect of these variables by other researchers (Eckey and Goldress, 1989; Ihom et al., 2012). Ihom et al. came up with a model equation for the prediction of green permeability as:

$$Y_{X1X2P} = 25.37 + 0.0114X_1 + 61.73X_2 \quad (1)$$

Where, Y_{X1X2P}, is the green permeability as a result of the interaction of the two variable factors, clay and water content. The authors said that this equation could be used to predict the green permeability of the clay-bonded sand all conditions been equal as those obtained in their work. This is no doubt a better way of studying green permeability since it took into consideration the interactive influence of the two variables.

The objective of this present work is to build upon what they did and to study the green permeability of a different sand mixture using multiple linear regression models for the prediction of green permeability values in this sand system and possibly compare it with earlier works.

MATERIALS AND METHODS

The materials used for this work were gotten from empirically generated data on various moulding sand systems at the Sand Testing Laboratory of the National Development Centre, Jos. This particular data was generated from green permeability tests on green moulds using River Benue sand, clay binder and moisture (H_2O). The main variables in the work were clay (binder) which was varied from 1.5 to 7% and moisture (water) which was varied from 2 to 8%. Other variables like mixing time, number of rams, sand (grain size and shape) were all kept constant. Table 1 shows the data that was used for this study.

Multiple regression model development

The basic two variable models (one dependent and one independent variable) is:

$$Y = a + bx \quad (2)$$

Which can be solved using the normal equations thus:

$$\sum Y = an + b\sum x \quad (3)$$

$$\sum XY = a\sum X + b\sum X^2 \quad (4)$$

From this can be developed models with more than two variables and this is illustrated below using a 3 variable model (one dependent and two independent variables, Y, X_1, and X_2):

$$Y = a + b_{1X_1} + b_{2\,X_2} \quad (5)$$

Which can be solved by the normal equations for a three variable model, as follows:

$$\sum Y = an + b_1\sum X_1 + b_2\sum X_2 \quad (6)$$

$$\sum X_1\,Y = a\sum X_1 + b_1\sum X_1^2 + b_2\sum X_1X_2 \quad (7)$$

$$\sum X_2\,Y = a\sum X_2 + b_1\sum X_1X_2 + b_2\sum X_2^2 \quad (8)$$

The line of best fit gives way to a plane of best fit, b_1 is the slope of the plane along the X_1 axis, b_2 is the slope along the X_2 axis and the plane cuts the Y axis at 'a'. The aim of adding to the simple two variable models is to improve the fit of the data. The closeness of fit is measured by the coefficient of multiple determination R^2 for which the general formula and a useful computational formula is given below:

$$R^2 = \frac{a\sum Y + b_1\sum X_{1Y} + b_2\sum X_{2Y} - \frac{(\sum Y)^2}{n}}{\sum Y^2 - \frac{(\sum Y)^2}{n}} \quad (9)$$

Table 2. Calculation of separate regressions with permeability as Y_P.

	Y_P	Y^2_P	X1	X^2_1	X_2	X^2_2	X_1Y_P	X_2Y_P	X_1X_2
1	310	96100	1.5	2.25	2	4	465	620	3.0
2	280	78400	2	4	3	9	560	840	6.0
3	300	90000	3	9	4	16	900	1200	12.0
4	320	102400	4	16	5	25	1280	1600	20.0
5	310	96100	5	25	6	36	1550	1860	30.0
6	300	90000	6	36	7	49	1800	2100	42.0
7	270	72900	7	49	8	64	1890	2160	56.0
∑	2090	625900	28.5	141.25	35	203	8445	10380	169.0

Table 2 gives the calculation of separate regressions with permeability as $Y_{P,}$ clay content as X_1 and moisture content as $X_{2.}$

For regression Y_P on X_1 (Clay content %)

$$b_{x1} = \frac{n\sum X_{1Y_P} - \sum X_1 \sum Y_P}{n\sum X_1^2 - (\sum X_1)^2} = \frac{7\,X8445 - 28.5 \times 2090}{7 \times 141.25 - (28.5)^2} = -2.55$$

$$a_{x1} = \frac{\sum Y_p}{n} - \frac{bX_1 \sum X_1}{n} = \frac{2090}{7} - \frac{-2.55\;x\;28.5}{7} = 308.95$$

The regression equation for the relationship of clay content and permeability value of the green mould is:

$$Y_{X_1} = a_{X_1} + b_{X_1} X_1 = 308.95 + (-2.55X_1) = 308.95 - 2.55X_1$$

$$Y_{X_1} = 308.95 - 2.55X_1 \quad (10)$$

The coefficient of correlation for this relationship is:

$$r_{X_1} = \frac{n\sum X_1 Y_P - \sum X_1 \sum Y_P}{\sqrt{n\sum X_1^2} - (\sum X_1)^2 \times \sqrt{n\sum Y_p^2 - (\sum Yp)^2}} \quad (11)$$

Substituting the values in Table 2 in equation 11, we have $r_{x1}^2 = 0.09$, that is, coefficient of determination for $Y_P : X_1$

For regression of Y_P on X_2 (% Moisture content)

$$b_{X_2} = \frac{n\sum X_2 Y_P - \sum X_2 \sum Y_P}{n\sum X_2^2 - (\sum X_2)^2} = \frac{7 \times 10380 - 35 \times 2090}{7\,X\,203 - (35)^2} = -2.5$$

$$a_{X2} = \frac{\sum Y_p}{n} - \frac{b_{X_2} \sum X_2}{n} = 298.57 - (-2.5\,X\,35) = 386.07$$

The regression equation for the relationship of moisture content with permeability value of the green mould is:

$$Y_{PX_1} = a_{X_2} + b_{X_2} X_2 = 386.07 + (-2.5X_2) = 386.07 - 2.5X_2 \quad (12)$$

The coefficient of correlation for this relationship is:

$$r_{x2} = \frac{n\sum X_2 Y_P - \sum X_2 \sum Y_P}{\sqrt{n\sum X_2^2} - (\sum X_2)^2 \times \sqrt{n\sum Y_p^2 - (\sum Yp)^2}} = \frac{-490}{\sqrt{196}\;X\;\sqrt{13200}} = -0.31$$

$r_{X_2}^2 = 0.0961,$ that is, coefficient of determination for $Y_P : X_2$

The multiple regression (Y_P : X_1 and X_2)

The multiple regression calculations are carried out using the three variable normal equations (6-7) with substitution with values from Table 2 and calculated values of a_{x1}, b_{x1}, a_{x2}, b_{x2} new set of equations are derived as follows:

$$2090 = 7a + 28.5b_1 + 35b_2 \quad (13)$$

$$8445 = 28.5a + 141.25b_1 + 169b_2 \quad (14)$$

$$10380 = 35a + 169b_1 + 203b_2 \quad (15)$$

Solving these three equations simultaneously gave:

$a = 341.58$
$b_1 = 47.53$
$b_2 = -47.33$

Now substituting the model equation for three variables in equation 5 this new model equation is obtained:

$$Y_P = 341.58 + 47.53X_1 - 47.33X_2 \quad (16)$$

This model equation can be used to predict the permeability of the green mould. For example what is the expected permeability of a green mould prepared with 8% clay and 8% moisture content.

$$Y_p = 341.48 + 47.53 \times 8 - 47.33 \times 8 = 343.18$$

When $X_1 = 3$, $X_2 = 4$, $Y_P = 341.58 + 47.53 X 3 – 47.33 X 4 = 294.85$

Coefficient of multiple determination, R^2

From equation 9,

$$R^2 = \frac{(341.58\ X\ 2090) + (47.53 \times 8445) + (-47.33\ X\ 10380) - \frac{(2090)^2}{7}}{625900 - \frac{(2090)^2}{7}} = -0.0035$$

RESULTS

The developed model equation for the combined influence of clay binder and moisture content variation on the green permeability of the green mould is:

$$Y_P = 341.58 + 47.53X_1 - 47.33X_2 \quad (17)$$

The various coefficient of correlations and coefficient of multiple determination can now be summarised and interpreted:

$r_{x1} = -0.30$
$r_{x2} = -0.31$
$R^2 = -0.0035$

DISCUSSION

r_{x1} indicates that about 30% of the variation in permeability is caused by variation in clay content this is not a major influence and the influence is negative which means that as the permeability is increasing the clay content is decreasing and vice versa.

r_{x2} equals - 0.31 means that 31% of the variations in green permeability are caused by variation in moisture content of the mould. This is a moderate influence and it is negative. The implication is that as moisture content increases the green permeability decreases. Why this is the case in later stages of the relationship the earlier stages show a direct relationship until the optimum point is reached as explained earlier (Jain, 2009).

R^2 equals - 0.0035 means that - 0.35% is the combined influence of the two variables clay and moisture content on the green permeability and it is in the negative direction. Which means any increase in the two variables will lead to a small decrease in permeability and vice versa. Going by the interpretation of the values of the coefficient of determination and correlation coefficients above the value of the coefficient of multiple determination is correct since the combined effects of the variables will cancel out but the balance will tilt towards a decrease in permeability.

Analysis juxtaposed existing theories on green permeability in sand moulds

According to Jain (2009), green permeability decreases with increase in clay content the relationship is inversely proportional; however, in the presence of ample moisture this relationship may not be drastic as when the moisture is low. From above, using multiple regression model on the other hand has shown that 30% of the variation in permeability is caused by variation in clay content percent and the influence is negative which means that as the clay content is increased the permeability decreases. This is in agreement with Jain. The inversely proportional theory still holds with the model. The influence of moisture on permeability is such that as moisture is increased permeability also increases until an optimum point or peak is reached where further increase in moisture results in decrease in permeability (Ihom et al., 2011; Ihom et al., 2006; Eckey and Goldress, 1989; Ihom et al., 2012). This relationship was confirmed by Jain for both round and angular shaped sands. The model result showed that 31% of the variation in permeability was due to moisture content variation. The influence of moisture on permeability was a negative influence which means that as moisture increases permeability decreases. The model have only succeeded in describing the later part of the relationship of moisture and green permeability, but failed to describe the early part of the relationship between moisture content increases with green permeability as observed by Jain, (2009), Ihom et al. (2011), Ihom et al. (2006), Eckey and Goldress (1989) and Ihom et al. (2012).

Clay content increase brings about decrease in green permeability while moisture content increase, initially brings increase in green permeability further increase however, brings about decrease in green permeability. The combined effect at the end of the day is decrease in green permeability. The model has it that 0.35% increase in green permeability is due to the combined effect of clay content and moisture content percent increase. The model is correct since the combined effect of the two variables does not encourage too much increase in green permeability (Jain, 2009; Ihom et al., 2006; Eckey and Goldress, 1989; Ihom et al., 2012).

The developed model for prediction of green permeability has taken into consideration the combined effect of the two variables; clay and moisture content percent on green permeability. From above analysis, we have seen that the relationship models (coefficients) have tried to fit into existing theories to a large extent. This implies that the prediction model in equation 16 can be used to predict or estimate green permeability in green moulds. The example above using 8% clay and 8% moisture gave a permeability value of 343.18 AFS Perm units. At 3% clay and 4% moisture the green permeability is 300 AFS Perm units and the calculated value using the model equation is 294.85 AFS Perm units, the model is therefore not a bad estimation tool for green permeability.

CONCLUSION

The study, Analysis and Prediction of Green Permeability Values in Sand moulds using Multiple Linear Regression Model, has been carried out and the following conclusions drawn:

1. The developed mathematical model can be used for a close estimation of green permeability in green sand moulds provided the same conditions (mixing time, ramming, green state) are the same as in this work.
2. The relationship models in the work made some good attempt in describing the relationship between green permeability and the variables clay and moisture content percent.
3. Existing theories on the relationship between green permeability and the two variables of clay and moisture content percent agree to a large extent with the multiple linear regression model approach used in the work.

ACKNOWLEDGEMENT

The authors hereby extend their sincere appreciation to the staff of the Sand Testing Laboratory of NMDC Jos who assisted with the generation of this data. We also acknowledge your effort for the enormous amount of data bank that now exists in that Laboratory on Nigerian foundry sands.

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