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Investigation of the accuracy of non-linear model in the study of green permeability of green sand moulds

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ABSTRACT

The study, "investigation of the accuracy of non linear model in the study of green permeability of green sand moulds" has been carried out. The nonlinear model used in the work is based on the hyperbolic function. The work used results generated empirically from the Sand Testing Laboratory of National Metallurgical Development Centre, Jos. Non-linear models were developed for the study and prediction of green permeability. Green permeability was the dependent variable while clay content and moisture content were the independent variables. The developed models were tested for accuracy both in terms of interpreting the relationship existing between green permeability and the two variables and in predicting green permeability. It was found that this model failed in terms of accuracy. A graphical version of the model was developed from the model equations and it was found that the non-linear model was completely different in shape and form from the graphical model of the green permeability generated empirically. While the later conformed to existing theories on permeability, the former did not. The study has clearly established the inaccuracy of nonlinear model based on hyperbolic function for prediction of green permeability and analysis of relationship between green permeability and the two variables of clay content and moisture content in green sand moulds.

Keywords: Accuracy, nonlinear models, permeability, moulds, investigation.

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INTRODUCTION

According to Asuquo and Ihom (2013), a model is any representation of reality and may be in graphical, physical or mathematical terms. Mathematical model is commonly used because it tries to show the working of the real world by means of mathematical symbols, equations and formulae (Asuquo and Ihom, 2013; Lucey, 2000).

Green permeability is an important property of green sand moulds. This is because of the role it plays in the formation of defects like pinholes and pores in solidifying cast metal. High permeability in moulds allows the escape of gases without entrapment in the solidifying molten metal. However, when the permeability is low the solidifying metal stands the risk of having defects arising from entrapped gases (Jain, 2009). This partly explains why a lot of work is done on studying the green permeability of moulds. Most of this work has been empirical-based, designs of single factor experiments of tests conducted on measurement of permeability in moulds (Ihom et al., 2006).

In recent times, a lot of interest has been generated in the study of green permeability using different models (Ihom et al., 2006; Eckey and Goldress, 1989). Models like simple linear regression, multiple linear regression and factorial approaches have been used by different researchers (Ihom et al., 2009; Zrimsek and Vingas, 1961; Chakoraborty and Dhindaw, 1979) in the investigation of this important property in green sand moulds. The authors have noted that most of the models

Table 1. Green permeability data with variation in clay and moisture content.

S/No	% Clay X_1	% Moisture X_2	Permeability Y_p (AFS perm unit)
1	1.5	2	310
2	2	3	280
3	3	4	300
4	4	5	320
5	5	6	310
6	6	7	300
7	7	8	270

used are linear in nature. The big problem that actually needs to be resolved is; can we say the influence of moisture and clay on permeability is linear? Of course not! Most of the literatures (Chakoraborty and Dhindaw, 1985; Ihom, 2012; Khanna, 2008; Brown, 1994) reviewed for this work have clearly shown that permeability increases with increase in moisture content until an optimum point is reached where additional increase in moisture content results in decrease in permeability. The graph of this variation is a curve and not a straight line. Increase in clay content of the green sand mould results in the decrease of permeability the steepness of the curve also depends on the amount of water in the moulding mixture. Here too the variation is not a straight line. The question again is how correct are linear models in the study and prediction of green permeability? The combined effect of moisture and clay content on permeability will definitely be unique since the shape of the curve at 2% moisture content is different from that of 4% moisture content (Heine et al., 1967). Complete study of green permeability will therefore have to involve the use of models that include multiple variables (Chakoraborty and Dhindaw, 1979; Chakoraborty and Dhindaw, 1985; Heine et al., 1967).

The objective of this study is to investigate the accuracy of non-linear model in the study of green permeability of green sand moulds.

MATERIALS AND METHODS

The materials used for this work were obtained from empirically generated data on various moulding sand systems at the sand testing laboratory of the National Development Centre, Jos. This particular data was generated from green permeability tests on green moulds using River Benue sand, clay binder and moisture (H_2O). Equipment like; electric permmeter, ramming machine, mixers were used for the test. The main variables in the work were clay which was varied from 1.5 to 7% and moisture which was varied from 2 to 8%. Other variables like mixing time, number of rams, sand (grain size and shape) were all kept constant. Table 1 shows the data that was used for this study.

Non-linear models

$$Y = a + \frac{b}{X} \quad (1)$$

Equation 1 is a typical hyperbolic function (model) where 'a' is intercept and a constant, b is also a constant representing the slope.

The values of a and b are calculated by reference to amended formulae:

$$b = \frac{n\Sigma\left(\frac{1}{X}\right)Y - \Sigma\left(\frac{1}{X}\right)Y}{n\,\Sigma\left(\frac{1}{X}\right)^2 - \left(\Sigma\frac{1}{X}\right)^2} \quad (2)$$

$$a = \frac{\Sigma Y}{n} - b\,\frac{\Sigma\left(\frac{1}{X}\right)}{n} \quad (3)$$

X_1 = Variation of Clay content (%), X_2 = Variation of Moisture content (%), and Y_P = Green permeability in AFS Perm units. (Table 2)

Calculating the value of b by using Equation 2 we have:

$$b = 17602.73$$

From Equation 3 we have:

$$a = -5384.6$$

Thus the hyperbolic function is:

$$Y_p = -5384.6 + \frac{17602.73}{X_1} \quad (4)$$

Table 3 indicates that all the predicted values using the nonlinear model in Equation 4 are way off the experimental values in the same table. The plot of the experimental values and the predicted values against $1/X_1$, is shown in Figures 1 and 2 so that we can visualize the shape of the plot. (Table 4).

Calculating the value of b using Equation 2, we have:

$$b = 29505.88$$

Table 2. Calculations for the Least Square Line of Best fit, $Y_P : X_1$.

S/No.	$1/X_1$ (inverse of clay content %)	Y_P	$(1/X_1)^2$	(1/X)Y
1	0.67	310	0.45	207.7
2	0.50	280	0.25	140.0
3	0.33	300	0.11	99.0
4	0.25	320	0.06	80.0
5	0.20	310	0.04	62.0
6	0.17	300	0.03	51.0
7	0.14	270	0.02	37.8
∑	2.26	2090	0.96	677.5

Table 3. Predicted values of green permeability using the non linear model (Equation 4).

$1/X_1$ (inverse of clay content %)	Y_P experimental values	Y_P predicted values
0.67	310	6350.55
0.50	280	3416.77
0.33	300	482.98
0.25	320	-983.92
0.20	310	-1864.05
0.17	300	-2450.81
0.14	270	-2869.92

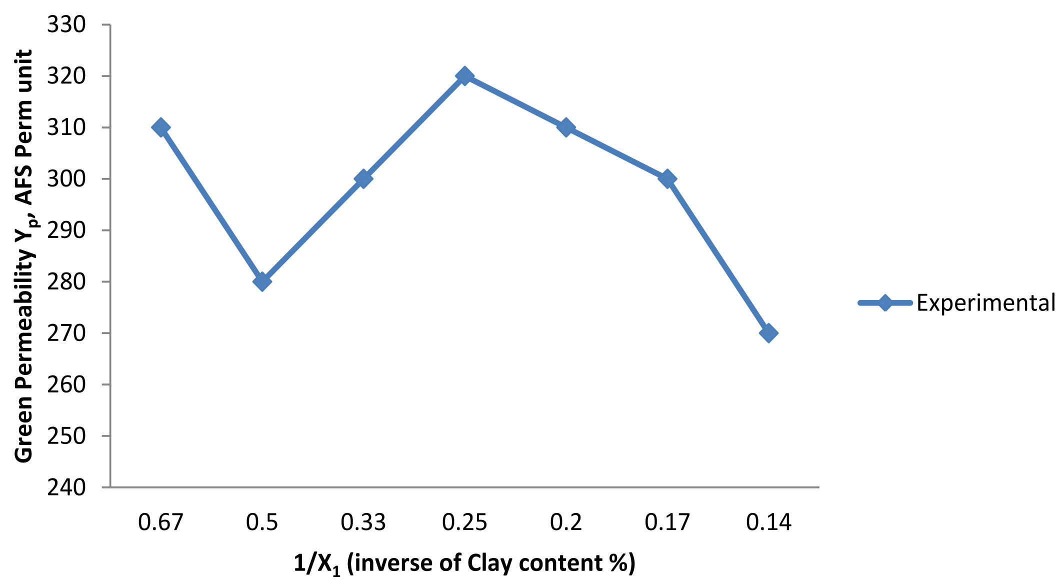

Figure 1. Experimental green permeability variation with inverse clay content percent.

From Equation 3, we have:

$$a = -6951.45$$

Thus the hyperbolic function is:

$$Y_p = -6951.45 + \frac{29505.88}{X_2} \quad (5)$$

Table 5 indicates that all the predicted values using the nonlinear model in Equation 5 are way off the experimental values in the same table. The plot of the experimental values and the predicted values against $1/X_2$ is shown in Figures 3 and 4, so that the shape of the plotted graph can be visualized.

RESULTS

The developed model for the relationship between green

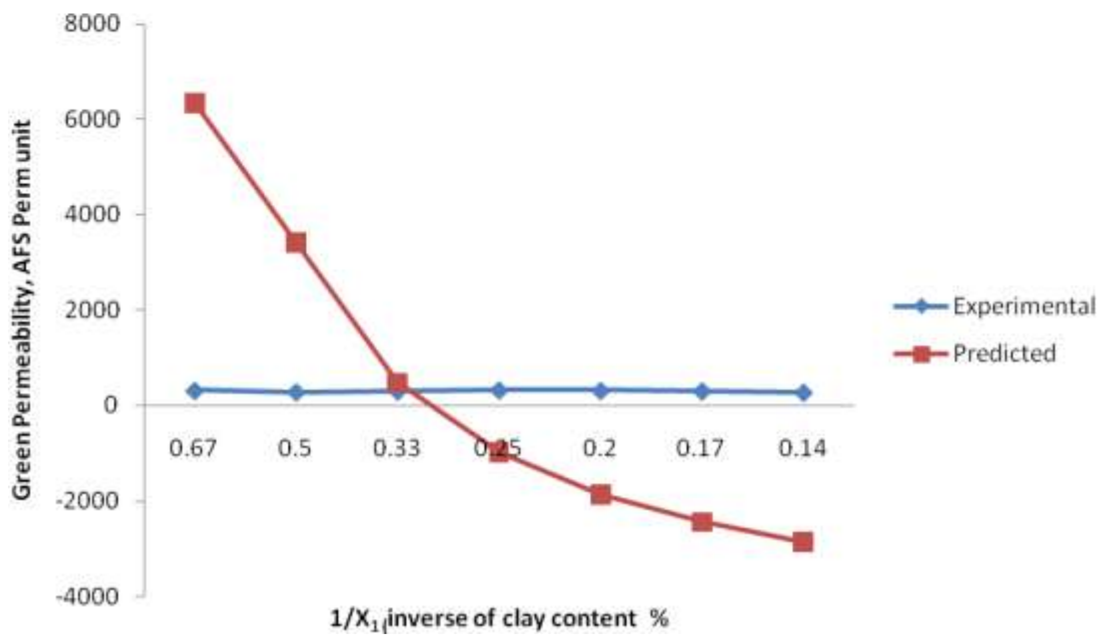

Figure 2. Experimental green permeability and predicted green permeability variation with inverse clay content percent.

Table 4. Calculations for the least squares of best fit using moisture content % Y_P : X_2.

S/No	$1/X_2$ (inverse of moisture content %)	Y_P	$(1/X_2)^2$	(1/X)Y
1	0.5	310	0.25	155
2	0.33	280	0.11	92.4
3	0.25	300	0.06	75
4	0.20	320	0.04	64
5	0.17	310	0.03	52.7
6	0.14	300	0.02	42
7	0.13	270	0.017	35.1
∑	1.72	2090	0.527	516.2

Table 5. Predicted values green permeability using the non linear model (equation 5).

$1/X_2$ (inverse of moisture content %)	Y_P experimental values	Y_P predicted values
0.50	310	7801.49
0.33	280	2883.84
0.25	300	425.02
0.20	320	-1050.27
0.17	310	-2033.80
0.14	300	-2736.32
0.13	270	-3263.22

permeability and clay content variation is:

$$Y_P = -5384.6 + \frac{17602.73}{X_1} \quad (4)$$

The developed model for the relationship between green permeability and moisture content variation is:

$$Y_P = -6951.45 + \frac{29505.88}{X_2} \quad (5)$$

Other results are shown in Tables 3 to 5 and Figures 1 to 4.

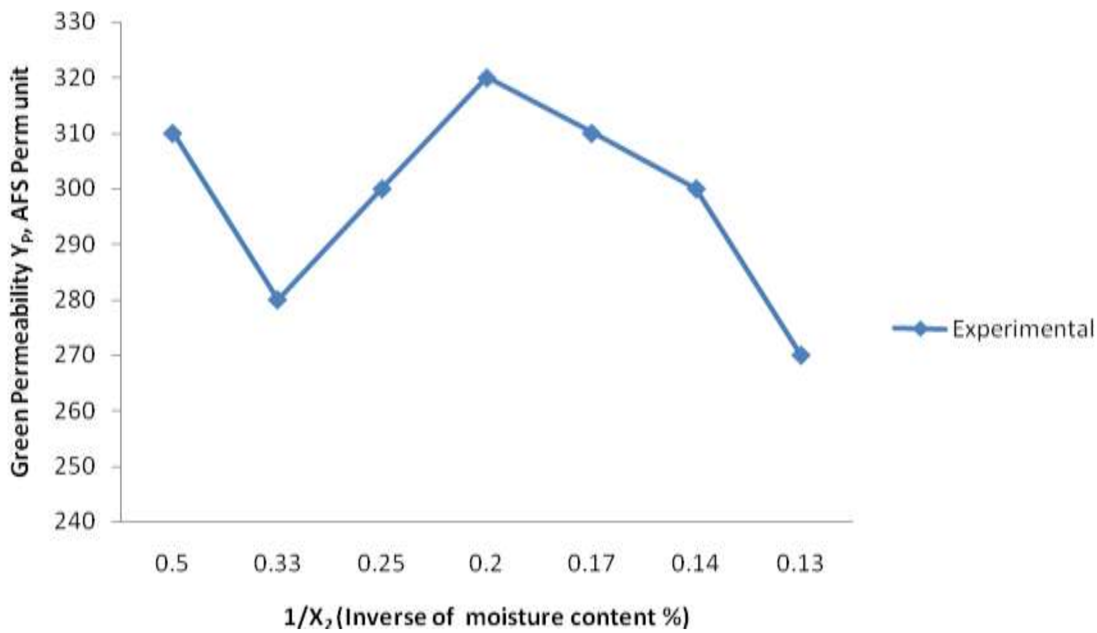

Figure 3. Experimental green permeability variation with inverse moisture content percent.

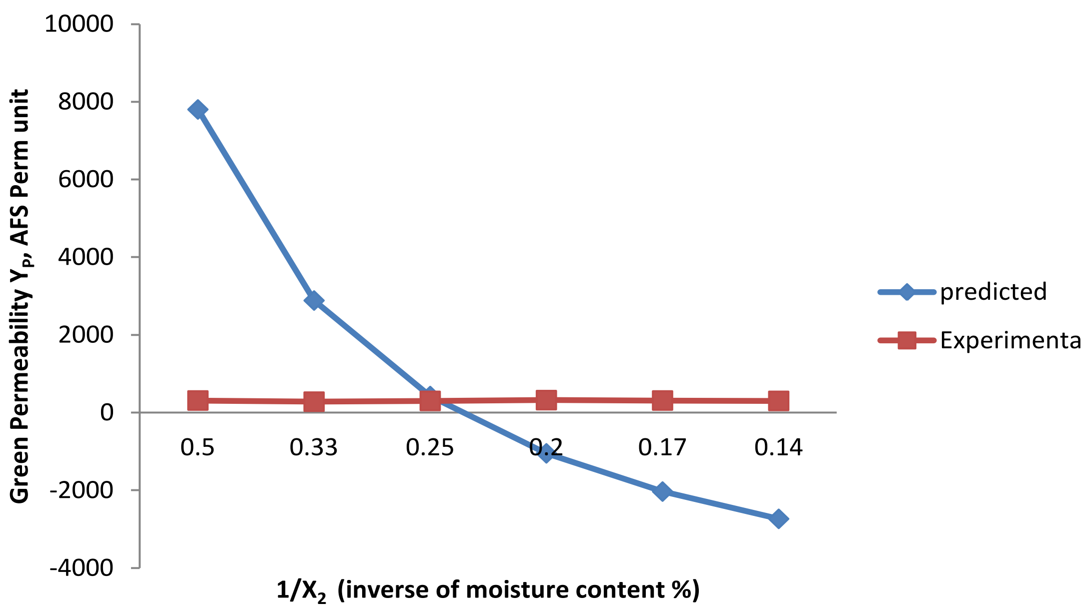

Figure 4. Experimental green permeability and predicted green permeability variation with inverse moisture content percent.

DISCUSSION

Analysis of the accuracy of the non-linear model (the hyperbolic function) in the study of green permeability of green sand moulds

Equation 4 is a non-linear model describing the relationship between green permeability and clay content. The model is indicating that as clay content increases green permeability is also increasing and as permeability increases clay content increases. This also can be seen in Figure 2 (the graph of the predicted values based on the model). The movement is in one direction. This is contrary to what has been observed by Jain (2009), who said “Increase in clay content of the green sand mould results in the decrease of permeability the steepness of the curve also depends on the amount of water in the moulding mixture. Here too the variation is not a straight line". The nonlinear model is not linear too. However, the calculated or predicted values of green permeability are way off different from the experimental values. Table 3 shows that the values are not close in any way; casting doubt on the accuracy of the nonlinear model. To further have an insight into this nonlinear model a plot of the experimental permeability values with inverse clay content variation was made separately in Figure 1 and then experimental permeability and predicted permeability with inverse variations in clay content on the same graph in Figure 2. Figure 1 shows that the relationship is not a linear one, but also not a perfect hyperbolic curve or function. The curve agrees with researchers who say that "increase in clay content of the green sand mould results in the decrease of permeability; the steepness of the curve also depends on the amount of water in the moulding mixture (Heine et al., 1967; Ihom et al., 2011; Ihom et al., 2012; Ihom et al., 2009). Looking at Figure 2, the reason for the inability of the nonlinear model to accurately predict the values of green permeability becomes clear. The nonlinear model is a perfect hyperbolic curve whereas the curve of the experimental permeability is not. Figure 2 also confirms why linear regression models have proven to be more reliable in predicting green permeability values than nonlinear models [Ihom et al., 2009; Zrimsek and Vingas, 1961; Chakoraborty and Dhindaw, 1979, 1985; Ihom et al., 2009). Ihom et al. came up with a model equation for the prediction of green permeability as:

$$Y_{X1X2P} = 25.37 + 0.0114X_1 + 61.73X_2 \quad (6)$$

This model was developed using Multiple Linear Regression Method and the model can predict green permeability to a very close range (Ihom et al., 2006).

Equation 5 is the nonlinear model describing the relationship between green permeability and moisture content of the green mould. The equation says that as moisture content increases green permeability increases. The model illustrates inverse moisture content variation with green permeability, according to the model as the inverse of moisture content is decreasing so also is the green permeability decreasing. This relationship is inadequate in describing the relationship between moisture content (the independent variable) and green permeability the dependent variable. Most of the literatures (Chakoraborty and Dhindaw, 1985; Ihom, 2012; Khanna, 2008; Brown, 1994; Heine et al., 1967; Ihom et al., 2011; Ihom et al., 2012; Ihom et al., 2009) reviewed for this work have clearly shown that permeability increases with increase in moisture content until an optimum point is reached where additional increase in moisture content results in decrease in permeability. The graph of this variation is a curve and not a straight line. Which of course the model equation too is not a straight line. A look at the values in Table 5 shows that the values of the predicted or calculated green permeability using the nonlinear model of Equation 5 are not the same with the values of the experimental green permeability neither are they close in value. Which means the model cannot be used to predict green permeability in green moulds. The plot of the experimental and predicted permeability in Table 5 is shown in Figures 3 and 4. In Figure 3, experimental green permeability is plotted against inverse of moisture content. The graphical model is a curve, falling, rising and reaching a peak where further increase in moisture only lead to a decrease in permeability. This agrees with literatures reviewed (Ihom et al., 2006; Chakoraborty and Dhindaw, 1985; Ihom, 2012; Khanna, 2008; Brown, 1994; Heine et al., 1967; Ihom et al., 2011; Ihom et al., 2012; Ihom et al., 2009). The graphical model of the predicted permeability in Figure 4 is not like that, it is a perfect hyperbolic function showing unidirectional decrease of both the independent variable (inverse moisture content) and the dependent variable (predicted green permeability). Figure 4 is a plot of both the experimental and predicted green permeability against inverse of moisture content the two curves are different in shape; while the predicted green permeability curve is a perfect hyperbolic curve, the experimental permeability curve is not. The predicted permeability curve indicates that as moisture content is increasing the green permeability is also increasing and vice versa. This is indicated by the continuous descent of the curve from left to right down to the negative side of the y- axis representing permeability. This model is way-off and not correct, since it did not agree with any of the works reviewed (Chakoraborty and Dhindaw, 1985; Ihom, 2012; Khanna, 2008; Brown, 1994; Heine et al., 1967; Ihom et al., 2011; Ihom et al., 2012; Ihom et al., 2009). The plot of the two nonlinear model equations; Equations 4 and 5 does not indicate that green permeability can be predicted using nonlinear model with hyperbolic function.

CONCLUSION

The Investigation of the accuracy of nonlinear model in the study of green permeability of green sand moulds has been investigated. The work has drawn the following conclusions:

1. Nonlinear models with hyperbolic functions have failed to adequately describe the relationship between green permeability and clay content and also with moisture content.
2. Nonlinear model could not correctly predict green permeability values. The accuracy of this model is therefore in doubt.
3. The nonlinear models are perfectly hyperbolic and generate values of permeability that are hyperbolic

function dependent.

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