Oscillatory magnetohydrodynamic (MHD) Stoke’s flow past a flat plate with induced magnetic field effects

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ABSTRACT

Oscillatory magnetohydrodynamic (MHD) Stoke’s flow of a viscous incompressible electrically conducting fluid past a flat plate in the presence of a transverse magnetic field has been investigated. In a fully ionized fluid, the effect of induced magnetic field is considered by applying Laplace transform method where the electric field cannot be ignored. None of the author has considered the effect of induced magnetic field on MHD flow system by employing Laplace transform method with the ignorance of an electric field. Since the MHD flow oscillates harmonically with time, a time varying electromagnetic field of sinusoidal in nature is subjected to a plasma flow of equal number of ions and electrons. Since the MHD flow is associated with a perfectly conducting wall, the magnetic field does not penetrate outside the flow region. Since the phase angle is characterized by a time varying electromagnetic field of sinusoidal in nature, the effect of induced magnetic field communicates a tendency of spiraling on MHD flow with increase in phase angle ($\omega \tau$). In varying time ($\tau$) the effect of induced magnetic field leads to a spiraling in nature. The situation reveals that the charged particle gyrates round the lines of force in a MHD flow field. Numerical results of velocity distributions, induced magnetic field distributions and the frictional shearing stress are depicted graphically.

Keywords: Oscillatory MHD flow, magnetic force, Laplace transform method, induced magnetic field, plasma state, phase angle.

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INTRODUCTION

Magnetohydrodynamic (MHD) flow has received wide attention in literature with reference to different conditions and configuration. The study of magnetohydrodynamics communicates a challenging approach to plasma science leading to an application of astrophysics and fluid engineering. An orientation of such study emerges the backbone of a fluid system with a decisive importance to analytical approach. Authors have been studied the effect of induced magnetic field on MHD channel flow system by applying analytical method. Mention may be made of their works of Gupta (1969), Denno and Fouad (1972), Mazumder et al. (1976), Ghosh (1991, 1993), Ghosh and Nandi (2000), Ghosh and Bhattacharjee (2000), Ghosh (2002), Ghosh et al. (2009), Beg et al. (2009) and Ghosh et al. (2010). All these studies have been taken into account to expedite MHD flow behaviour. Nevertheless, MHD flow past a flat plate by considering the effect of induced magnetic field has been studied by Ghosh and Pop (2006) and Ghosh et al. (2010). Although radiative heat transfer aspect on hydrodynamic flow past a flat plate was considered by many authors by employing Laplace transform method where the effect of induced magnetic field becomes insignificant. This has been studied by Fujii and Imura (1972), Lewandowski (1991), Raptis and Perdikis (2003), Ghosh and Pop (2007), Ghosh (2007) and Beg et al. (2011). Magnetohydrodynamic flow with radiative heat transfer past a flat plate has been taken into account with the study of Helmy (1999) and Ghosh et al. (2015) with reference to Laplace transform method where the effect of induced magnetic field becomes ignored. All these studies have been extended by many authors with different aspects of the flow medium. However, several investigations have been made by the researchers with different conditions and configurations. Authors have studied magnetohydrodynamic flow in a different situation.
by neglecting induced magnetic field. These are studied by Kumar and Gupta (2012), Mabood and Ibrahim (2016), Nikodijevic et al. (2011), Prasad et al. (2013), Ramana Murthy and Srinivas (2013), Ramesh and Devakar (2015), Tetbirt et al. (2016) and Ghosh (2018, 2019). A literature survey reveals that none of the author has considered the effect of induced magnetic field by employing Laplace transform method. Mathematical methods on MHD flow past a flat plate pose formidable mathematical challenge to exert its influence on Laplace transform method where the effects of induced magnetic field become significant.

The aim of present investigation is to develop mathematical competence to expose the effect of induced magnetic field on MHD Stokke’s flow past an oscillating flat plate by applying Laplace transform method. In a fully ionized fluid, the effect of induced magnetic field becomes predominant over the entire MHD flow system. In the presence of electric field the effect of polarization becomes significant with reference to \( \nabla \times E = -\frac{\partial B}{\partial t} \). In a fully ionized fluid, the plate is considered perfectly conducting so that the magnetic field does not penetrate outside the region of space and flow becomes a plasma state where the equal number of ions and electrons is balanced by the magnetic field. The flow of electron discharge from the plasma state of flow is subjected to a kinetic energy of the system. Indeed, in a fully ionized fluid, plasma fusion interacts with hot electron in the presence of a magnetic field in a controllable region when the kinetic energy is transformed into heat.

**FORMULATION OF THE PROBLEM AND ITS SOLUTION**

Consider an unsteady flow of a viscous incompressible electrically conducting fluid occupying a semi-infinite region of space bounded by an infinite oscillating plate. An infinitely extended flat plate takes place of harmonic oscillations with a constant velocity \( U \) in its own plane. Since the plate oscillates harmonically with time, a transverse magnetic field is imposed perpendicular to the plate to exert its influence on a pendulum so that the magnetic field does not penetrate outside the region and an oscillatory motion appears to be a significant manner in the presence of a magnetic field. It seems to be understood that a uniform magnetic flux density \( B_0 \) is applied perpendicular to the plane. Since the plate lies along \( x' \)- axis, the coordinate system reveals to \( x' \)-axis along the plate and \( y' \)-axis is normal to it. Since the pressure is constant in the whole space, an infinitely extended plate along \( x' \)-axis it reveals that all physical quantities except pressure will be functions of \( y' \) and \( t \) only.

The MHD Equation of motion reads:

\[
\frac{\partial q}{\partial t} = -\frac{1}{\rho} \nabla p + u \nabla^2 q + \frac{1}{\rho \mu_0} (\nabla \times B) \times B
\]  

(1)

The conservation of mass leads to the Equation of continuity:

\[
\nabla \cdot q = 0
\]  

(2)

The solenoidal relation that the magnetic field produces no source or sink reads:

\[
\nabla \cdot B = 0
\]  

(3)

Since the motion becomes unsteady, the Maxwell Equation becomes:

\[
\nabla \times E = -\frac{\partial B}{\partial t}
\]  

(4)

The other Maxwell Equations are:

\[
\nabla \times H = J
\]

\[
\nabla \cdot J = 0
\]

(5)  

(6)

where \( B = \mu_0 H \)

The Ohm’s law for a moving conductor:

\[
J = \sigma [E + q \times B]
\]  

(7)

The magnetic induction Equation becomes:
\[ \frac{\partial B}{\partial t} = \text{curl} (q \times B) + \frac{1}{\mu_e} \nabla^2 B \]  (8)

Equation 7 is closely resemble to Equation 8 so that Equations 7 and 8 are identical.

q, B, E and J are, respectively, the velocity vector, magnetic field vector, electric field and current density vector. Further, \( \rho, \mu_e, \nu, \sigma, t \) and \( p \) are the fluid density, magnetic permeability, kinematic coefficient of viscosity, electrical conductivity, time and constant pressure.

According to the Geometry of the problem, Equations (1) to (8) are in agreement with following Equations reads:

The MHD Equation of motion:
\[
\frac{\partial u'}{\partial t'} = v \frac{\partial^2 u'}{\partial y'^2} + \frac{1}{\mu_e} (\nabla \times B') \times B'
\]  (9)

The magnetic induction Equation becomes:
\[
\frac{\partial B_x'}{\partial t'} = \text{curl} (u' \times B_x') + \frac{1}{\sigma} \nabla^2 B_x'
\]  (10)

The initial and the boundary conditions for velocity become:

\( u' = 0 \) for all \( y' \geq 0 \) and \( t' \leq 0 \),
\( u' = U \cos \omega t' \) at \( y' = 0 \) for \( t' > 0 \),
\( u' \to 0 \) as \( y' \to \infty \) for \( t' > 0 \)  (11)

Since the plate is considered perfectly conducting, the following magnetic boundary conditions are:
\[
\begin{align*}
\frac{d B_x'}{d y'} &= 0 \text{ at } y' = 0, \\
\frac{d B_x'}{d y'} &= 0 \text{ at } y \to \infty, \\
B_x' \to 0 \text{ as } y \to \infty
\end{align*}
\]  (12)

Introducing dimensionless variables:
\[
\begin{align*}
\tilde{u} &= \frac{u'}{u}, \quad \tau = \frac{t' u^2}{v}, \quad y = \frac{y' u}{v}, \quad \omega = \frac{\omega' u}{v}, \quad B_x = \frac{B_x'}{\sigma \mu_e v_0}
\end{align*}
\]  (13)

and \( M^2 = \frac{\sigma \mu_0 v}{\rho u^2} \) is the Hartmann number.

The Equations (9) to (10) can be represented in a dimensionless form read:
\[
\begin{align*}
\frac{\partial \tilde{u}}{\partial \tau} &= \frac{\partial^2 \tilde{u}}{\partial y^2} + M^2 \frac{\partial \tilde{B}_x}{\partial y}, \\
\frac{\partial \tilde{B}_x}{\partial \tau} &= \frac{\partial \tilde{u}}{\partial y} + \frac{\partial^2 \tilde{B}_x}{\partial y^2},
\end{align*}
\]  (14)

where \( P_m = \sigma \mu_e v \) is the magnetic Prandtl number.

The corresponding velocity boundary conditions are:
\[
\begin{align*}
u &= 0 \text{ for all } y \geq 0 \text{ and } \tau \leq 0, \\
u &= \cos \omega \tau \text{ at } y = 0 \text{ for } \tau > 0, \\
u \to 0 \text{ as } y \to \infty \text{ for } \tau > 0
\end{align*}
\]  (16)

and the corresponding magnetic boundary conditions become:
\[
\begin{align*}
\frac{d \tilde{B}_x}{d y} &= 0 \text{ at } y = 0, \\
\frac{d \tilde{B}_x}{d y} \to \infty \text{ as } y \to \infty, \\
\tilde{B}_x \to 0 \text{ as } y \to \infty
\end{align*}
\]  (17)
With reference to Laplace transform method, the following Equations 14 to 15 subject to boundary conditions (16) and (17) take the form:

$$s u^* = \frac{\partial^2 u^*}{\partial y^2} + M^2 \frac{\partial^2 \theta^*}{\partial y^2}$$

(18)

$$P_m s B_x^* = \frac{\partial^2 B_x^*}{\partial y^2} + \frac{\partial u^*}{\partial y}$$

(19)

The corresponding velocity boundary conditions are:

$$u^* = 0 \text{ for all } y \geq 0 \text{ and } \tau \leq 0,$$

$$u^* = \frac{y}{y^2 + \omega^2} \text{ at } y = 0 \text{ for } \tau > 0,$$

(20)

$$u^* \to 0 \text{ as } y \to \infty \text{ for } \tau > 0$$

The corresponding magnetic boundary conditions are:

$$\frac{d B_x^*}{d y} = 0 \text{ at } y = 0,$$

$$\frac{d B_x^*}{d y} \to 0 \text{ at } y \to \infty,$$

(21)

$$B_x^* \to 0 \text{ at } y \to \infty$$

To analyze Equation 19, the following Equation takes the form:

$$\frac{\partial B_x^*}{\partial y} = P_m s \left[ B_x^* y - \int y \frac{d B_x^*}{d y} dy \right] - u^* + C_1$$

(22)

Where $C_1$ is arbitrary constant.

Proceeding in this way, Equations 18 to 22 subject to boundary conditions (20) – (21) the solution of velocity distributions and induced magnetic field distributions can be obtained by applying inverse Laplace transform method such as:

$$u(y, \tau) = \frac{1}{4} \left[ 1 + \frac{M^2}{M^2 - i \omega} \right] e^{-i \omega \tau} \left[ e^{y \sqrt{M^2 - i \omega}}.erfc \left( \frac{\sqrt{y^2 + \tau(M^2 - i \omega)}}{2\sqrt{\tau}} \right) + e^{-y \sqrt{M^2 - i \omega}}.erfc \left( \frac{-\sqrt{y^2 + \tau(M^2 - i \omega)}}{2\sqrt{\tau}} \right) \right] + \frac{1}{4} \left[ 1 + \frac{M^2}{M^2 + i \omega} \right] e^{i \omega \tau} \left[ e^{y \sqrt{M^2 + i \omega}}.erfc \left( \frac{\sqrt{y^2 + \tau(M^2 + i \omega)}}{2\sqrt{\tau}} \right) + e^{-y \sqrt{M^2 + i \omega}}.erfc \left( \frac{-\sqrt{y^2 + \tau(M^2 + i \omega)}}{2\sqrt{\tau}} \right) \right]$$

(23)

Induced magnetic field distribution $B_x(y, \tau)$

$$B_x(y, \tau) = \frac{i \omega}{4 \sqrt{M^2 - \omega} \sqrt{M^2 + \omega}} \left[ e^{y \sqrt{M^2 + \omega} + i \omega} \right].erfc \left( \frac{\sqrt{y^2 + \tau(M^2 + \omega)}}{2\sqrt{\tau}} \right) - e^{-y \sqrt{M^2 + \omega}}.erfc \left( \frac{-\sqrt{y^2 + \tau(M^2 + \omega)}}{2\sqrt{\tau}} \right)$$

(24)

Frictional shearing stress $\tau_x$ at the plate $y = 0$ becomes:

$$\tau_x \equiv \frac{d u}{d y} \bigg|_{y=0} = \frac{1}{2} \left[ 1 - \frac{M^2}{M^2 - i \omega} \right] e^{-i \omega \tau} \left[ -\frac{1}{\sqrt{\pi \tau}} e^{-\sqrt{M^2 - i \omega} y} \left( 1 - erfc \left( -\sqrt{\tau(M^2 - i \omega)} \right) \right) \right] + \frac{1}{2} \left( 1 - \frac{M^2}{M^2 + i \omega} \right) e^{i \omega \tau} \left[ -\frac{1}{\sqrt{\pi \tau}} e^{-\sqrt{M^2 + i \omega} y} \right]$$

(25)
The solutions [23-25] are in agreement with the solutions obtained by Ghosh (2019) when \( \omega(frequency) = 0 \) and \( \omega \tau (\text{phase angle}) = 0 \)

**RESULTS AND DISCUSSION**

The graphical discussions in relevance to a physical interpretation has been made with arbitrary values of \( M^2 \) (magnetic force), \( \omega \) (frequency), \( \omega \tau \) (phase angle) and \( \tau \) (time) in Figures 1 to 11. In Figures 1 to 8, since the magnetic field does not penetrate outside the region of flow the effects of velocity and induced magnetic field on oscillatory MHD flow behave on stabilizing influence on the flow region and all the profiles converge near the flow region with reference to the perfectly conducting plate. Figure 1 show that an increase in \( M^2 \) (magnetic force) leads to fall the velocity. This happens in the case of an MHD flow to distort the magnetic lines of force in the presence of a Lorentz force. It is evident from Figure 2 that the velocity increases with an increase in \( \omega \) (frequency). An increase in \( \omega \) (frequency) on the velocity field exerts its influence of a magnetic force to accelerate the MHD flow field. Figure 3 reveals that the velocity decreases with an increase in \( \omega \tau \) (Phase angle). Since velocity exerted by the angular displacement in the presence of a strong magnetic field so that the velocity is reduced by increasing \( \omega \tau \) (Phase angle). In this situation, in a fully ionized flow, the flow of electron behaves in a suppressed manner to balance the MHD flow medium that leads to stabilize near the flow situation. This happens in the case of an oscillatory MHD flow in a controllable region where the equal number of ions and electrons becomes predominant. It is evident from Figure 4 that the velocity increases with an increase in \( \tau \) (time). On increasing \( \tau \) (time) the flow velocity increases with a decisive importance to a magnetic force to accelerate the MHD flow field.

Figure 5 reveals that the induced magnetic field decreases with an increase in \( M^2 \) (magnetic force). Since induced magnetic field produced by the motion of an electrically conducting fluid the effect of induced magnetic field tends to destabilize on the MHD flow system so that magnetic field does not penetrate outside the flow region. Figure 6 demonstrates that the induced magnetic field increases with an increase in \( \omega \) (frequency). In a strong magnetic field, the effect of induced magnetic field leads to a progressive influence on the MHD flow field to increase the frequency in a kinetic energy of the system.

![Figure 1. Velocity profile when varying \( M^2 \).](image)

This happens near the plate before elapse at a distance of the MHD flow region. Figure 7 reveals to an oscillatory character on the effect of induced magnetic field near the plate and converge all the profiles of induced magnetic
field in a plane of flow. With an increase in \( \omega \tau \) (Phase angle) the effect of induced magnetic field is spiraling in nature near the plate. Since the magnetic field is strong enough, the charged particles gyrate round the lines of
force near the plane of flow. In an oscillatory MHD flow, the effect of induced magnetic field is subjected to a phase angle to represent ionized plasma leading to a kinetic energy of the system. In Figure 8 it is noticed that the effect of induced magnetic field increases close to the plane of flow with an increase in $\tau$ (time) while it decreases near the flow region with an increase in $\tau$ (time), whereas it becomes an oscillatory character just
before elapse from the distance. In this situation, there is a tendency of spiralling the curve so that electron moves spiralling in nature round the magnetic lines of force with increase in $\tau$ (time) due to a kinetic energy of the system.

The effect of frictional shearing stress at the plate are plotted against $M^2$ (magnetic force) with an arbitrary value of $\omega$ (frequency), $\omega \tau$ (phase angle) and $\tau$ (time). Figure 9 shows that the frictional shear stress increases with
increase in either $M^2$ (magnetic force) or $\omega$ (frequency). There arises a separation close to the plate with increase in either $M^2$ or $\omega$. In the case of Figure 9, frictional drag has an active influence to the plane of flow adjacent to the plate. In Figure 10, the frictional shearing stress increases with an increase in either $M^2$ or $\omega \tau$ but there exists flow separation in the range $10 \leq M^2 \leq 16$. There arises a viscous drag reducing influence on MHD flow
Figure 10. Frictional shearing stress when varying $\omega \tau$.

Figure 11. Frictional shearing stress when varying time $\tau$.

field near the plate due to flow separation at the leading edge of the plate. Figure 11 reveals that the frictional shearing stress increases with an increase in either $M^2$ or $\tau$. There exists flow separation when the numerical values of $\tau$ become 0.2 to 0.25 and separation does not occur when the value of $\tau$ is 0.3 and above. Here the viscous drag plays a significant role in determining the MHD flow at the leading edge of the plate.
The physical significance of the problem leading to a time varying electromagnetic field of a sinusoidal in nature exerts its influence of an induced magnetic field so that the charged particle gyrates round the lines of force in a magnetohydrodynamic flow field. This situation reveals that the effect of induced magnetic field is spiralling in nature near the plate. This implies that the emission of hot electrons spiralling in a magnetic field is a synchrotron radiation when the kinetic energy is transformed into heat.

CONCLUSION

Oscillatory MHD flow past a flat plate by employing Laplace transform method has been studied. In a fully ionized fluid, induced magnetic field produced by the motion of an electrically conducting fluid becomes significant where electric field cannot be ignored. In a perfectly conducting plate, magnetic field does not penetrate outside the flow region. All the authors have neglected the effect of induced magnetic field by applying Laplace transform method so that the simplifications of all these problems are made in the absence of electric field. In a fully ionized fluid, the representation of a plasma flow leads to an equal number of ions and electrons so that the charged particle gyrates round the magnetic lines force in a MHD flow system. In a time varying electromagnetic field of sinusoidal in nature, the phase angle exerted by the induced magnetic field in determining the MHD flow behaviour. There arises a tendency of spiralling in nature on induced magnetic field with the increase of either phase angle or time. Indeed, the induced magnetic field has a pronounced effect on MHD flow field.

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Additional comments

Oscillatory magnetohydrodynamic (MHD) Stoke’s flow with induced magnetic field effects is a new investigation on plasma science under the influence of induced magnetic field by applying Laplace transform method which has not received attention in literature. The effects of induced magnetic field on a perfectly conducting plate become significant so that the magnetic field does not penetrate outside the region.

REFERENCES


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