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Graphical and mathematical modeling of tumor shape

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ABSTRACT

Researchers have, for sometimes now, been working on the idea of tumor modeling for the purposes of determining tumor growth pattern, size, and size prediction using lots of formal approaches. In this paper, a different approach to modeling tumor is presented. It is a post-segmentation approach that involves quantification of tumor present in all tumorous MRI slice of a patient resulting in the graphical transformation of brain tumor 2D shapes into a continuous shape on Cartesian plane. A number of such transforms are then collected in reasonable number to form a single model of tumor shape. The graphical model of tumor shapes is then mathematically modeled to enable computer simulation and quick numerical quantification of tumor volume. The model would be useful for quick understanding of tumor spread into different segment of the brain during IGS planning, and in computational oncology.

Keywords: Tumor modeling, MRI slice, brain tumor, tumor shape model, mathematical model, tumor spread, IGS planning, computational oncology.

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INTRODUCTION

Tumor quantification is the process of finding the actual size (volume) of tumor that is present in patient image record. Earlier attempts at quantifying tumor had focused on developing tumor growth model consequent upon which shape could be modeled and tumor size determined or predicted. Such works include the Malthusian and Gompertzian tumor growth curves (Peter and Semmle, 2004), and tumor doubling time by Mehrara et al. (2007). Other works in tumor modeling include Kundel and Dean (2002) who suggested the use of images to identify tumor extent, and in addition to that, many classical works, such as Soltanian-Zadeh and Windham (2002), have proposed a method called ‘partial volume model’ based on voxel information, and Gorniak et al. (2002) using fusion technique, which is aimed at determining tumor extent including several biopsies methods. Iftekharuddin (2005) reported lots of fractal analysis methods of identifying tumor boundary, and authors like Landini et al. (2008), John (2008), ter Haar Romeny (2008), Park et al. (2005), Goldenberg et al. (2005) and Liew and Yan (2005), have reported worthy segmentation methods whose eventual target is determination of tumor extent.

The approach adopted in this work is based on the suggestion made by Kundel and Dean (2002), that is, use of image in determining tumor size. The method in this paper is essentially a post-segmentation operation whereby tumor regions from MRI slices have been segmented (Figure 1) using any of the existing and medical image inclined segmentation method. The first step therefore is the computation of tumor area using iterative spatial sectoring (ISS) method (Aboaba et al., 2012). This is followed by plot on the Cartesian plane that resulted is tumor area average versus slice thickness from where the mathematical model is evolved. This approach is occasioned by the fact that what is easily available in terms of brain tumor data is the MRI slices which shows varying degree of tumor from slice to slice, and the uniformly spaced slice thickness. Therefore, the method provides a way of making optimal use of this

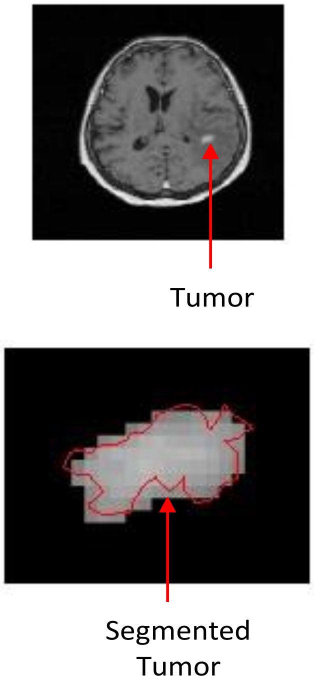

Figure 1. Tumor location in a Slice (above) and Segmented Tumor (below).

information. The mathematical model aimed could be integrated to estimate tumor volume.

TUMOR AREA DETERMINATION USING ISS

The iterative spatial sectoring (ISS) method developed for calculating area of irregular close shape stemmed from the Simpson’s rule of approximate integral. It involves the division of tumor shapes into smaller sector of equal angle and different radii as shown in Figure 2. The radii and sector angle are then used as input parameters into the ISS equation to determine the area of individual sector and consequently area of the irregular shape. A further reduction in sector angle is done with a new set of radii measured (Figure 3) to determine the area of the close shape a second time, and the process continues (Figure 4) until convergence is reached. The idea of finding area of irregular close shape by iterative spatial sectoring is based on its high accuracy with few numbers of iteration and its capability to set error margin as reported in Aboaba et al. (2012).

The initial sector angle is calculated using Equation 1

$$Initial\ Sector\ Angle(\theta_{x=1}) = \frac{\left|\left[\frac{A}{C}360^{\circ} - 360^{\circ}\right]\right|}{10} \dots\dots\dots (1)$$

Area of subsequent sectors in the iterative process is governed by Equation 2.

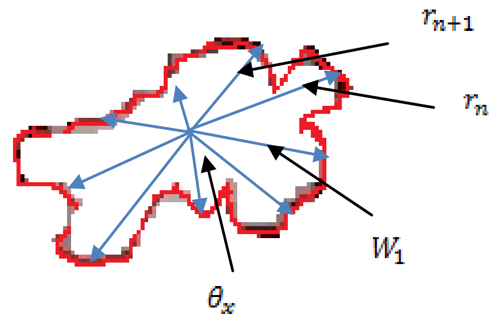

Figure 2. Segmented tumor shape divided into sectors and radii labeled from r_n to W_1.

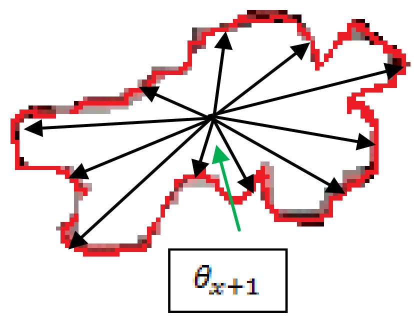

Figure 3. Segmented tumor shape with further reduction in sector angle (first iteration).

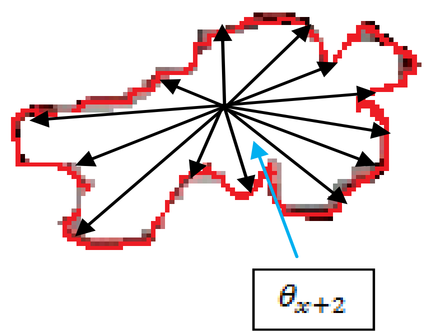

Figure 4. Segmented tumor shape with further reduction in sector angle (second iteration).

$$Sector\ Angle(\theta_{x+i}) = \frac{\left|\left[\frac{A}{C}360^{\circ} - 360^{\circ}\right]\right|}{10.(i+1)} \dots\dots\dots (2)$$

Where θ_x is the sector angle, i is current iteration, A is area of the circle exterior to the shape, and C is the area of the circle interior to the shape (Aboaba et al., 2012). Therefore, the area of individual sector of the close shape is:

$$Area\ of\ a\ Sector\ (A_s) = \frac{\theta_x}{360^o}\pi r_n . r_{n+1} \dots \dots \dots (3)$$

Hence, the area of irregular close shape (R_s) for the first iteration is:

$$\sum A_s = \sum_{n=1}^{W_1-1} (\frac{\pi\theta_x}{360^o})_n . r_n r_{n+1} + (\frac{\pi\theta_x}{360^o})_{W_1} . r_{W_1} r_n \dots \dots \dots (4)$$

Where r_n and w_1 stand for the numerical value of the first and last radii, respectively. Thus, for subsequent iteration the area of the irregular shape (R_{ss}) is:

$$\sum A_s = \sum_{n=1}^{W_1-1} (\frac{\pi\theta_{x+i}}{360^o})_n . r_n r_{n+1} + (\frac{\pi\theta_{x+i}}{360^o})_{W_1} . r_{W_1} r_n \dots \dots \dots (5)$$

Finally, the iteration stops at convergence, that is, when the area calculated from current iteration is equal or almost equal to the one in previous calculation. This is achieved when:

$$|R_{ss} - R_s| = 0 \dots \dots \dots (6)$$

Hence, tumor area is said to be the value of the last iteration, and the same procedure is carried out on the next slice till all the tumorous slices are processed. Furthermore, this procedure is repeated for all patients' image record to have substantial number of area versus slice thickness data. The error margin could be set by equating the left hand side of Equation 6 to a certain value other than zero.

PLOTTING SEGMENTATION DATA

Since the objective is to find a suitable mathematical model for the data, a simple way to achieving that is to find the average of all data points with respect to individual slice thickness. This gives a single set of data, that is, slice thickness and average tumor area as shown in Table 1. This is then plotted as shown in Figure 5.

CURVE MODELING

Since the curve is a quadratic curve, it is modeled using a second degree polynomial equation:

$$y = ax^2 + bx + c \dots \dots \dots (7a)$$

Solving this equation means solving for a and b while the rest (y, x and c) are found from the graph. In order to

Table 1. Slice thickness and average tumor area.

SN	Slice thickness (ST)	Average tumor area
1	0	0.00004
2	0.002	0.00012
3	0.004	0.00024
4	0.006	0.00030
5	0.008	0.00025
6	0.010	0.00032
7	0.012	0.00030
8	0.014	0.00025
9	0.016	0.00025
10	0.018	0.00022
11	0.020	0.00014
12	0.022	0.00007

determine a and b two equation are needed to be solved simultaneously; hence, a minimum of two points are needed on the curve to give two sets of x and y. in this wise, we pick the vertex point and any other point and noting that c is the curve intercept with the y-axis.

$$Vertex\ point = (x_1 = 0.009, y_1 = 0.00032)$$
$$Second\ point = (x_2 = 0.018, y_2 = 0.00023)$$
$$y - intercept(c) = 0.00004$$

Solving after parameter magnification by 1000 we have:

$$a = -\frac{3.33}{1458} = -0.0023$$
$$b = \frac{0.465}{9} = 0.0517$$

and $c = 0.04$

Therefore the curve model equation is:

$$y = -2.3 \times 10^{-6}x^2 + 51.7 \times 10^{-6}x + 40 \times 10^{-6} \dots \dots \dots (7b)$$

CONCLUSION

Equation 7b represents the eventual mathematical model of the tumor shape. As a preliminary step, three patients' records are used to graph the model curve, and based on its quadratic shape; a second degree polynomial is used to model the shape. The ISS used enable accurate determination of tumor area from individual tumor slices. The graph in Figure 5 is a simplified alternative way of looking at brain tumor pattern by allowing a full-length view of tumor progression in just one graphical display. If the population sample is increased, it could be used as a model for estimating brain tumor growth pattern, and

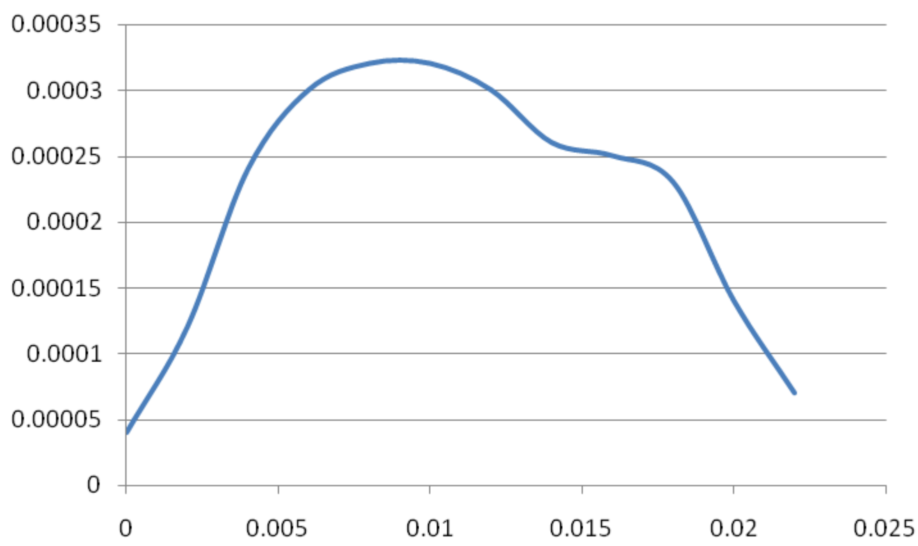

Figure 5. Plot of average tumor area versus slice thickness.

volume. Also, useful in image guided surgery (IGS) planning and intervention.

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