

# Theoretical mathematical model of non-invasive flow quantification for future implementation in routine clinical MRI

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## ABSTRACT

Automated push-button medical analysis using quantitative multiparametric standard scales will offer the possibility of efficient fast medical diagnosis in a precise and accurate manner. Magnetic Resonance Imaging (MRI) offers complex geometrico-physicochemical analysis of the physio-anatomy of interest. Most imaging protocols for the quantitative multiparametric MRI have already been implemented clinically, but flow assessment still remains a problem, especially when evaluated without contrast agents. A simple method for flow quantification using MRI without contrast agents is described in this study. The proposed method has the potential of non-invasive flow quantification, produced offline from MRI images acquired in less than one minute.

**Keywords:** MRI, multiparametric medical diagnosis, *in vivo* flow quantification, medical image processing, non-invasive imaging.

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## INTRODUCTION

Complex geometrico-physicochemical analysis of the physio-anatomy of interest is needed to understand disease mechanisms, detect, stage or assess disease and evaluate efficacy of new or existing therapies (Miller et al., 2014). Among the existing imaging techniques, MRI offers the most complex multiparametric geometrico-physicochemical information of the physio-anatomy of interest. The information includes: lengths or thicknesses (Fanea and Fagan, 2012), areas (Feczko et al., 2009), volumes (Feczko et al., 2009), relaxation times (Fanea and Fagan, 2012; Bojorquez et al., 2017), diffusion coefficients and fractional volumes of the diffusing components detected (Dong et al., 2004; Le Bihan, 2014), magnetization transfer ratios and the protein concentration (Gupta, 2002; Oreja-Guevara et al., 2006; Henkelman et al., 2001), magnetic susceptibility and iron or calcification concentrations (Duyn, 2013; Wang and Liu, 2015; Liu et al., 2015), and shear stiffnesses in elastography (Yin et al., 2016; Mariappan et al., 2010;

Venkatesh and Ehman, 2015; Hiscox et al., 2016). More recently, magnetic resonance spectroscopy has been implemented clinically to analyze the complex biochemical composition of the anatomy imaged (Soares and Law, 2009). All these parameters can be assessed without contrast agents (Fanea and Fagan, 2012; Calamante et al., 1999). Since 1990, several mathematical models have been developed (Fanea et al., 2012; Sourbrone and Buckley, 2013) and implemented in routine clinical MRI for flow quantification using contrast agents for MRI. More recent developments focused on flow quantification using arterial spin labeling techniques for MRI without contrast agents. However, these techniques are still difficult to implement in routine clinical MRI due to their decreased sensitivity (Grade et al., 2015; Amukotuwa et al., 2016; Haller et al., 2016; Calamante et al., 1999). An extremely simple and ready for clinical implementation method of flow quantification using MRI without contrast agents is proposed in this study.

## Mathematical model

The theoretical model assesses quantitatively flow based on the MRI signal intensity differences between corresponding anatomical regions in control/normal (C/N) and disease-affected (DA) images. A relative-to-normal global flow coefficient:  $f$  is calculated from this model and disease can be detected for  $f$  values other than 0.

If the spin density,  $M_0$  is assessed from a proton density (PD) image (Brown et al., 2014; Bernstein et al., 2004) and  $T_1$  and  $T_2$  of the region analyzed are known or calculated using the scenarios proposed in the literature (Deoni et al., 2005; Fanea and Sfrangeu, 2011), flow can be quantified using two MRI images. One image needs to be acquired from a disease-suspected subject and the other one from a C/N subject; unless a standard database already exists for the C/N image where flow can be assessed simply by acquiring an MRI image of the DA subject.

## METHODOLOGY

An MRI protocol eliminating or significantly reducing all other effects with exception of the  $T_1$ ,  $T_2$ , and flow can be used to acquire these images. In this situation, the signal intensity changes can be quantified based on the general  $T_1$  and  $T_2$  dependent magnetization ( $M$ ):

$$M = C \times M_0 \quad (1)$$

where  $C$  is the constant determined by the acquisition parameters and the  $T_1$  and  $T_2$  relaxation times.

The signal intensity of MRI images depends on the main acquisition parameters used: echo time (TE), repetition time (TR), flip angle (FA) and the physicochemical properties of the region imaged: the PD quantified by  $M_0$ , and the relaxometry quantified by  $T_1$  and  $T_2$  (Brown et al., 2014; Bernstein et al., 2004).

If flow differences exist between the two corresponding C/N and DA regions, the difference ( $\Delta$ ) of  $M$  between these two regions will be greater or less than 0. In this particular situation,  $\Delta$  depends on the:  $T_1$ ,  $T_2$  relaxation times, and the relative-to-normal global flow effects over the image acquisition time, TR. The relative-to-normal global flow effect can be quantified using the relative-to-normal flow coefficient:  $f$ . The difference between  $M$  in the corresponding DA and C/N regions can be expressed using Equation 1 and the relative-to-normal global flow effect on the initial spin density,  $M_0$ :

$$M^{DA} - M^{C/N} = C^{DA} \times M_0^{DA} - C^{C/N} \times M_0^{C/N} + f/(\lambda \times TR) \times (M_0^{DA} - M_0^{C/N}) \quad (2)$$

where  $\lambda$  represents the blood-to-tissue partition coefficient. Its values range from 0.77 to 1.04 (Herscovitch and Raichle, 1985), while a more general mean value of 0.9 ml/g is accepted in many studies (Roberts et al., 1996).

## RESULTS

$M_0/M$  and two more abbreviations (AB1) and (AB2) will be used to express  $f$ :

$$\Delta_0 = M_0^{DA} - M_0^{C/N} \quad (AB1)$$

$$\Delta = M^{DA} - M^{C/N} \quad (AB2)$$

For simplicity,  $M_0^{DA}/M_0^{C/N}$  can be expressed using (AB1):

$$M_0^{C/N} \times (M_0^{DA}/M_0^{C/N} - 1) = \Delta_0 \quad (AB1.1)$$

$$M_0^{DA}/M_0^{C/N} - 1 = \Delta_0/M_0^{C/N} \quad (AB1.2)$$

$$M_0^{DA}/M_0^{C/N} = \Delta_0/M_0^{C/N} + 1 \quad (AB1.3)$$

The expression of  $\Delta$  is obtained by introducing Equation 1, (AB1), (AB2), and (AB1.3) in Equation 2:

$$\Delta = f/(\lambda \times TR) \times \Delta_0 + (C^{C/N} \times M_0^{C/N}) \times [(C^{DA}/C^{C/N}) \times (M_0^{DA}/M_0^{C/N}) - 1] \quad (3)$$

Rearranging the terms in Equation 3:

$$f/(\lambda \times TR) \times \Delta_0 = \Delta - (C^{C/N} \times M_0^{C/N}) \times [(C^{DA}/C^{C/N}) \times (\Delta_0/M_0^{C/N} + 1) - 1] \quad (4)$$

f can be expressed in ml/g/ms:

$$f = (\lambda \times TR) \times \{\Delta/\Delta_0 - (C^{C/N} \times M_0^{C/N}/\Delta_0) \times [(C^{DA}/C^{C/N}) \times (\Delta_0/M_0^{C/N} + 1) - 1]\} \quad (5)$$

$f_{100g}^{\min}$  can also be introduced to quantify flow in ml of fluid / 100 g tissue / min:

$$f_{100g}^{\min} = [10^{(-2)}]/6 \times f \text{ [ml/ 100 g /min]} \quad (6)$$

### ***f calculation using MRI images***

The particular expression of f is also presented for two cases involving the most often used MRI imaging acquisition protocols: classical and spoiled gradient echo.

In classical MRI, M depends on the physicochemical parameters of the region imaged:  $M_0$ ,  $T_1$ ,  $T_2$  and the TE, and TR image acquisition parameters, while for spoiled gradient echo acquisition protocols, FA is also involved, as expressed in Equations 7 and 8 (Brown et al., 2014; Bernstein et al., 2004).

$$M = \begin{cases} M_0 \times (1-E1) \times E2 \text{ (7): classical} \\ M_0 \times [\sin FA \times (1-E1) \times E2]/(1-E1\cos FA) \text{ (8): spoiled gradient echo} \end{cases} \quad \text{MRI acquisition protocols.}$$

The following two abbreviations were used in Equations 7 and 8:

$$E1 = \exp(-TR/T_1) \text{ (AB3),}$$

$$E2 = \exp(-TE/T_2) \text{ (AB4).}$$

From equations 1 and 7, and equations 1 and 8, C can be expressed for the classical and spoiled gradient echo MRI protocols, respectively:

$$C = \begin{cases} (1-E1) \times E2 \text{ (9), classical} \\ [\sin FA \times (1-E1) \times E2]/(1-E1\cos FA) \text{ (10), spoiled gradient echo} \end{cases} \quad \text{MRI acquisition protocols.}$$

If a classical or spoiled gradient echo image acquisition protocol is used, then the expressions of C in equations (9) and (10), respectively can be used to replace C in Equation 5.

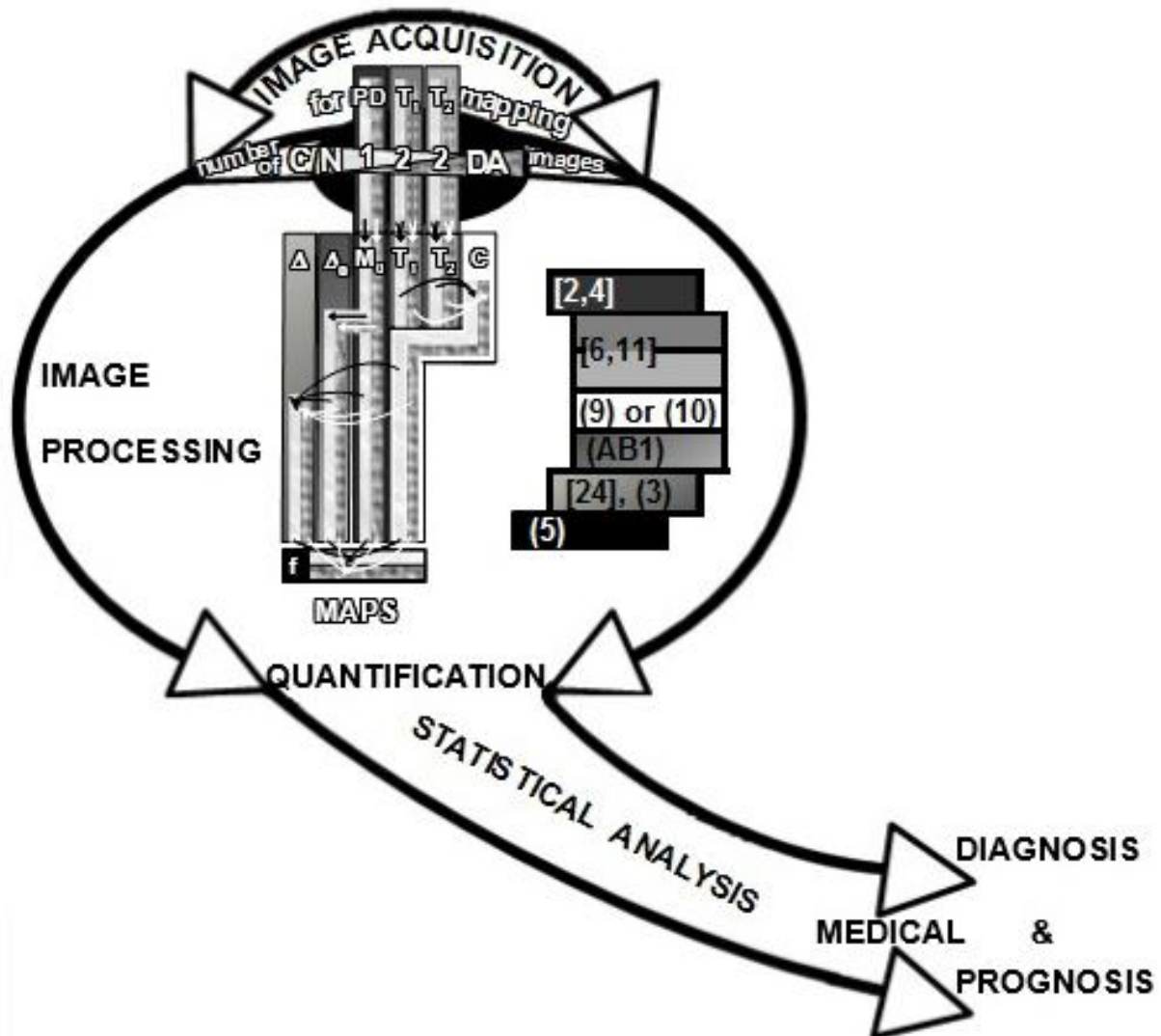
Details of the texture analysis based algorithm (Talu, 2012) used for the f calculation are presented in Figure 1.

The  $T_1$  and  $T_2$  relaxation times for the C/N and DA regions in Equation 5 can be calculated using the protocols presented in the literature (Deoni et al., 2005; Fanea and Sfrangeu, 2011). TR, TE, and FA in Equations 7 and 8 are known as the parameters set up in the MRI protocol for image acquisition. In MRI studies, a  $\lambda$  value of 0.9 ml/g is usually used (Roberts et al., 1996). The f value can, then be calculated by simply measuring the signal intensities in the two corresponding regions in the C/N and DA images (Figure 2). This allows the term  $\Delta$  to be calculated as the difference between the signal intensity in the two corresponding regions. Then two more PD images of the C/N and DA subjects need to be acquired to assess  $M_0$  and  $\Delta_0$  in the two corresponding C/N and DA regions.  $M_0$  is calculated by simply measuring the signal intensity in the specific C/N or DA region on the corresponding PD images.  $\Delta_0$  represents the difference of the signal intensity measured on the C/N and the corresponding DA regions on the PD images.

## **DISCUSSION**

The potential of MRI for the non-invasive flow quantification in acquisition times of up to 1 min is demonstrated in this study using a theoretical mathematical model. The mathematical model for flow calculation can be applied offline, but future software

developments will allow for flow quantification at the end of image acquisition. If flow charts are set up for C/N, then images need to be acquired only from the DA subjects. Even when the flow coefficients are calculated offline, these can give important information on the pathophysiology, together with the already clinically implemented multiparametric MRI techniques: relaxation



**Figure 1.** Texture analysis for flow quantification. Ten images: five of a C/N and five of a DA subject are acquired for PD (Brown et al., 2014; Bernstein et al., 2004), T<sub>1</sub> and T<sub>2</sub> (Deoni et al., 2005; Fanea and Sfrangeu, 2011) mapping. If the M<sub>0</sub>, T<sub>1</sub> and T<sub>2</sub> maps already exist, only two images need to be acquired: one from the C/N and one from the DA subject. If a database for the C/N already exists, then only the image of the DA subject needs to be acquired. The image(s) is(are) then processed and: Δ - (Roberts et al., 1996), Equation 3-, Δ<sub>0</sub> - abbreviation (AB1), M<sub>0</sub> - (Brown et al., 2014; Bernstein et al., 2004)-, T<sub>1</sub>, T<sub>2</sub> - (Deoni et al., 2005; Fanea and Sfrangeu, 2011)-, C - Equation 9 or 10- and f - Equation 5 - maps of the C/N and DA subjects are obtained. Flow is quantified in each region of interest and then statistically analysed based on Equations 3 and 5 for medical prognosis and diagnosis.

[2,4] = [Bernstein et al. 2004, Brown et al. 2014]

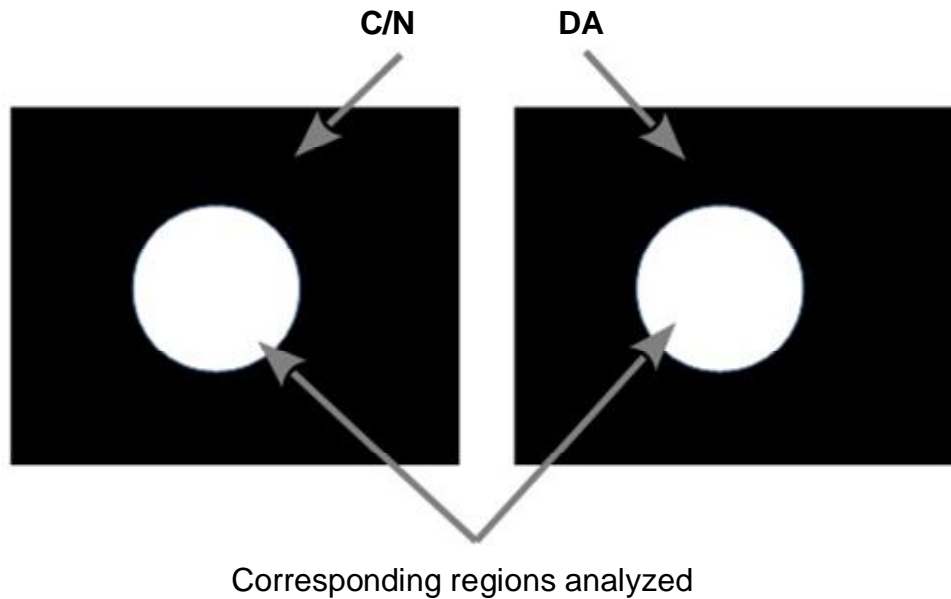
[6,11] = [Deoni et al. 2005, Fanea and Sfrangeu 2011]

[24] = [Roberts et al. 1996]

times (Fanea and Fagan, 2012; Bojorquez et al., 2017), diffusion (Dong et al., 2004; Le Bihan, 2014), magnetization transfer ratios (Gupta, 2002; Oreja-Guevara et al., 2006; Henkelman et al., 2001), magnetic susceptibility (Duyn, 2013; Wang and Liu, 2015; Liu et al., 2015), lengths (Fanea and Fagan, 2012; Feczko et al., 2009), areas (Feczko et al., 2009), volumes (Feczko et al., 2009), concentrations (Dong et al., 2004; Le Bihan, 2014; Gupta, 2002; Oreja-Guevara et al., 2006;

Henkelman et al., 2001; Duyn, 2013; Wang and Liu, 2015; Liu et al., 2015) and shear stiffnesses (Yin et al., 2016; Mariappan et al., 2010; Venkatesh and Ehman, 2015; Hiscox et al., 2016). The flow coefficients will contribute to the standard scale that can be built up for push-button automatic medical diagnosis. More importantly, introducing the flow quantification in the routine clinical MRI, detection of visually unperceived pathological changes could be possible, and therapies

Flow quantification using two MRI images acquired using exactly the same imaging protocol from the following two subjects:



**Figure 2.** Flow assessment strategy relative to a C/N region. PD,  $T_1$  and  $T_2$  maps from a C/N and DA subject are required for flow assessment in a region of interest (white circle). Acquisition of two more images (black squares), using exactly the same imaging protocol: one from a C/N (left black square) and the other one from a DA (right black square) subject, is also required. Flow is then assessed from these maps and images using the texture analysis described in Figure 1.

could be applied immediately and more efficiently by the medical staff. This may contribute to the reduction of the existing socioeconomic burden caused by many diseases.

More studies are needed to: evaluate the clinical implementation of the method proposed in this study, build the C/N flow standard scales, and assess the potential of this technique for disease detection, staging, assessment or for therapy and surgical evaluations.

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