A genetic algorithm approach to maximize crop yields and sustain soil fertility

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ABSTRACT

In order to increase crop production both for now and in future, conscious efforts need to be made towards sustaining soil fertility. One of the challenges facing farmers is how to increase crop yield and sustain soil fertility. Due to the multidimensional aspect of the challenge and, as a consequence, the huge set of potential solutions, field experiments are not well-suited for their choice and assessment. The objective of this study is to formulate, simulate and evaluate a genetic algorithm based model to maximize crop yields and sustain soil fertility. This study develops a nonlinear mixed-integer programming model to solve the maximization of crop yield problem with sustaining soil fertility. As it is an NP-hard problem, a genetic algorithms approach is proposed to determine the crop yield maximization while sustaining soil fertility. The soil fertility depends on several interrelated factors which have their respective determinants. Numerical analysis shows the effectiveness of the proposed method to deal with such a kind of complicated problem. The study identifies bio-physical, technical (including managerial) factors influencing soil fertility in the course of crop production. The regression result equally shows that for every hundred percent change in soil fertility, holding other factors constant, there is a substantial change in the crop yield. The findings of this study revealed that it is possible to scientifically appropriate all factors on sustainable basis (physical – such as soil nutrients, technical and managerial) with maximum crop yield achieved. On the basis of the findings, it is recommended that in order to have optimal utilization of resources on sustainable level, conscious efforts should be made by agricultural extension experts to ensure that farmers adopt management practices required for maximum yield.

Keywords: Genetic algorithm, crop yield, soil fertility, maximization, sustainability.

INTRODUCTION

Agriculture is changing rapidly: the market is globalised, society is concerned with environmental issues, people face food crises, energy crises call for bio-fuel production and legislative changes are made at global and local scales. Society desires development of new sustainable agricultural systems. Cropping systems, which can be seen as “a set of management procedures applied to a given, uniformly treated area” (Sebillotte, 1990), are the centre piece of such sustainable agricultural systems. In essence, they are complex systems involving many interactions between different biophysical (including environment factors) and technical (including managerial and social aspects) components.

With global population expected to exceed 9 billion by 2050, agricultural production would need to grow globally by 70 percent over the same period to feed this population. The need to feed more people puts greater pressure on crop production and the resource base upon which it depends. This is exacerbated by the additional pressures of coping with an increasingly degraded environment, uncertainties arising from climate change and other stressors such as increasing urbanization and volatile food prices. Further complicating this situation is that the global community must meet this increasing food demand in a world where ecosystem resilience is compromised, land resources available for agricultural
expansion are limited and soil fertility keeps depleting on regular basis. With land scarcity, crop production intensification rather than area expansion becomes the primary option available. Well-managed soil fertility is essential for ensuring a healthy resource base on which to intensify sustainably, to ensure that enough food is produced from now until 2050 – and beyond (FAO, 2009).

At the moment, even in the developing countries, the demand for land is extremely high and human population is on the increase. A large chunk of land, particularly, in cosmopolitan cities is being for residential purpose with few portions left for farming. Many farmers are having difficulty in increasing their farms’ crop yields as well as sustaining the fertility of the soil. This is owing to the fact that available lands are subjected to regular cultivation in order to meet increasingly high demands for farm products. The incessant cultivation of farmlands, however, reduces the soil fertility which, in turn, hampers the expected farm output.

The underpinning scientific and biological principles for improving soil health, managing pollination or controlling pest populations – incorporated in farming practices – show that yields can be increased through the sustainable management of soil fertility. Here, the role of farmers as custodians of biodiversity and as ecosystem managers is vital. At local levels, farming practices, approaches or technologies based on the management of biological processes that provide essential ecosystem goods and services can be applied to produce higher crop yields and optimize input use while maintaining or enhancing ecosystem [a community of living organisms (plants, animals and microbes) in conjunction with the nonliving components of their environment (things like air, water and mineral soil), interacting as a system] (MEAD, 2002).

Soil is a medium where plant grows and a base to apply plant nutrients. Hence improving and maintaining the fertility of the soil is crucial in agriculture. For farmers, feeding the crop means feeding the soil. Only a fertile soil can yield healthy crops and it is the most important resource of every farm. It is also noted that the maximization of crop yields is accompanied by depletion of soil fertility which in return reduces farm output, that is, poor agricultural output. This fact was also confirmed by Sanchez and Leakey (1997) in their work when they claimed that declining soil fertility is closely linked to productivity and has been identified as one of the root causes of declining per capita food production.

To increase future food production through intensification, however, conscious efforts need to be made towards sustaining soil fertility which is an integral part of the ecosystem. As a matter of fact, the goal of sustainably increasing crop production will not be achieved without improving, as well as, sustaining soil fertility too. The challenge most farmers are having is how to increase crop production or crop yield and as well maintain soil fertility. This study intends to show the possibility of increasing crop production and maintaining soil fertility with the aid of Genetic Algorithm. It intends to provide farmers with the technique to use in combining crop production with soil fertility. Inasmuch as farmers are making efforts to increase crop yields, the level of soil fertility that will guarantee the expected crop production should be, if not enhanced, maintained.

The importance of this study lies in the fact that it will provide farmers with a formula that guarantees simultaneous increase of their crop yields and the sustenance of soil fertility. With this study, the farmers will find lasting solution to the problem of increasing crop production without depleting the soil fertility that makes the increase in crop production possible.

**METHODOLOGY**

A range of options exist for good farm management practices, approaches and technologies that are based on biological processes. Examples include: conservation agriculture; integrated plant nutrient management; integrated pest management; and pollination management. These farm management practices are being increasingly used to achieve sustainable crop production intensification which has a key role in feeding the world, today and in the future.

Designing sustainable cropping systems is a complex multi-factorial decision problem (Sadok et al., 2009) that needs the conception, framing, building and assessment of proposed cropping systems. Due to the multidimensional aspect of the problem and, as a consequence, the huge set of potential solutions, field experiments are not well-suited for their choice and assessment. Given that sustainability encompasses economic, social and environmental dimensions, these dimensions are usually evaluated through different tools based on diverse multi-criteria decision-aid methods (Sadok et al., 2009).

The term “Maximization” indicates an optimization-related issue. Basically, optimization methods are searching procedures. They involve evaluating a myriad of possible problem solutions in order to find the best or optimal one. The trouble, of course, is that there are many possible solutions to search. That is, exhaustive search of all possibilities is simply inconceivable. This is why traditional optimization methods, such as ‘linear programming’ (Cheung and Auger, 1976) and ‘non-linear programming’, seek to reduce the number of solutions to be searched – they trim the ‘search space’. They rule out vast swathes of possible solutions because of the constraints that always surround any problem. They then search only the better solutions amongst those remaining.

This leads again to the problem of having to evaluate all possible solutions – an impossible task. In other words, conventional optimization and exhaustive search are, frequently, simply impractical. On this basis, an ‘Evolutionary Computing’ known as the ‘GENETIC ALGORITHM’ will be most appropriate. It concedes that finding the demonstrably optimal solution is impossible - it can only provide solutions that will keep getting better and approaching the universal optimal solution. In general, Genetic Algorithms can further be understood as an “intelligent” probabilistic search algorithm which is effective on such kind of complicated crop yield optimization problem (Felix et al., 2005; Zhou et al., 2002). Therefore, a model through hybrid genetic algorithm approach is developed with a local search mechanism to give out an optimal or near optimal solution of the problem.

Modelling itself provides a logical procedure for predicting
process outcomes in circumstances other than those that have been observed. Decision modelling aims to determine the optimal decision, define the trade-offs between different outcomes that are inherent in a range of decisions or predict the probable decisions that will be taken by farmers in a range of practical circumstances. Such models encapsulate knowledge of how a system is constructed of interacting processes and how each process works. They often combine experimental observations, expert knowledge and logic. In the physical world, models are frequently very precise and allow us, for example, to send probes to the moons of Jupiter. In the biological world not only are processes less well understood, often because they are made up of many sub-processes, but also the systems themselves are stochastic.

Modelling to aid decision making in sustainable agriculture does not require description of all elements in fine detail—the approach needs to be tailored for the purpose. Relatively simple descriptions of specific processes are suﬃcient if the processes are known to respond to a limited subset of external conditions, or if other unmodelled effects can be dealt with through appropriate adjustments to accommodate drift or errors. Early attempts at decision support systems in agriculture, such as Pro-Plant (Frahm et al., 1991), relied purely on expert knowledge to instruct the user in what to do. Pro-Plant Expert continues to function as an expert advisory system and covers a range of crops, pests and diseases. PC-Plant Protection, developed in Denmark (Murali et al., 1999), also uses expert scoring rules and covers control of weeds, pests and diseases in wheat, with an emphasis on reducing chemical use. EPIPRE in The Netherlands (Zadoks, 1981; Rijstdijk, 1983) used empirical models to relate observed disease levels to probable losses, but use of the system has now declined as farmers have become educated about the meaning of observations. Predictive modelling of the outcomes resulting from actions enables a person to make a better decision.

The methods to achieve this range from education/training so that operators better understand the consequences of their actions, through analytical studies and reports which provide the decision maker with measures of the eﬀect of various options, to computer-based decision support systems that use the models interactively to suggest the best decisions to the operator. Modelling decisions for these systems needs to combine a probabilistic approach to the range of possible outcomes with a deterministic description. The probabilistic approach could use stochastic modelling techniques (Sells, 1996), but for systems studies, direct application of probability modelling techniques to repeated simulations is more likely. The deterministic approach will generally describe component processes as logical relations or will use the fact that the overall system, the sum of the parts, often behaves in a fairly predictable way. Optimization is a powerful adjunct to predictive modelling for both the user and the modeller. In principle, its aim is to provide the farmer with the best decision. In this process, it is a very powerful test of the accuracy and completeness of a system model and, by association, of the expert knowledge. In addition, these techniques can be applied to a range of studies.

In addition, farming system models provide the means to assess the implications for optimal crop yields and optimal soil fertility. Model outputs increase understanding of how strategic decisions, by farmer or regulator, affect system performance. This study intends to provide farmers with a decision model that makes crop yield maximization, as well as, the sustenance of soil fertility possible.

From the discussion above and based on the research work of Min et al. (2005), this study develops a nonlinear mixed-integer programming model to solve the maximization of crop yield problem with sustaining soil fertility. As it is an NP-hard problem, a Genetic Algorithms approach is proposed to determine the crop yield maximization while sustaining soil fertility. The soil fertility depends on several interrelated factors and these factors have their respective determinants.

Following the iteration process, the overall solution of the proposed Genetic Algorithm is outlined below:

Initialization ( );
Soil fertility (SOIL_COMP)
For (gen = 1; gen<=MAX_YIELD; gen++)
Crossover ( );
Mutation ( );
Climbing ( );
Selection ( );
}
Output ( );

This study heavily relied on Figure 1 which was adapted from “Multi-attribute Assessment of the Sustainability of Cropping systems” (MASC) model to perform ex-ante assessments (Sadok et al., 2009) which was generated with the use of a computer programme for multi-attribute decision making called DEXi. It is aimed at interactive development of qualitative multi-attribute decision models and the evaluation of options. This type of modeling is useful for supporting complex decision-making tasks, where there is a need to select a particular option from a set of possible ones so as to satisfy the goals of the decision maker. A multi-attribute model is a hierarchical structure that represents the decomposition of the decision problem into sub-problems, which are smaller, less complex and possibly easier to solve than the complete problem.

Figure 1 shows some specific soil fertility factors and their relationship with crop yields. The study is however not unaware that in agriculture, that there are countless numbers of factors responsible for soil fertility.

Model formulation and specification

A model that captures the principal objective of this study is hereby specified with possible assumptions as follows:

i) Maximizing crop yield depends on the soil structure and soil fertility of the farmland.

ii) Crop yield is being influenced, not only by soil fertility but also by myriads of other important factors such as water use, climatic condition, pests and diseases, weed competition, social factors to mention but a few.

iii) The determinants of the crop yields are influenced by their respective exogenous variables.

iv) Soil fertility, on the other hand, depends on factors such as soil depth, availability of water, drainage, aeration, pH, mineral composition, organic matter and soil organism.

Crop yield model

The model reflects factors responsible for crop yield maximization aside from the soil fertility factor. Hence the model specification between the dependent and independent variables is stated as:

\[ \text{CRY} = f(\text{SOF, WTU, CLI, PAD, WEC, WST, VAC, LAU}) \]

And the regression form of the model is written as:

\[
\text{CRY} = \alpha_1 + \alpha_2 \text{SOF} + \alpha_3 \text{WTU} + \alpha_4 \text{CLI} + \alpha_5 \text{PAD} + \alpha_6 \text{WEC} + \alpha_7 \text{WST} + \alpha_8 \text{VAC} + \alpha_9 \text{LAU} + \mu
\]

(1)

Where, \(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7\) and \(\alpha_9\) are parameters of the model. The variables are:
Figure 1. Interrelationship between crop yields and soil fertility.
The full programming approach is presented. The stages involved are as follows:

**Genetic operation**

Genetic operation was used to alter the genetic composition of chromosomes or individuals. For the genetic representation, both crossover and mutation operation are adopted to make the exploitation and exploration searching in the evolutionary process.

**Crossover operation**

Crossover operated on two chromosomes at a time and generated offspring by combining both chromosomes' features. In particular, one-point crossover was used to improve all individuals in each generation.

The study randomly uses 68 chromosomes. It should be noted that the combination of two chromosomes at a time generates an offspring. Here, a probabilistic transition rule is applied on each chromosome to create a population of chromosomes. At the randomly selected point, a chromosome from the first parent combined with three chromosomes of the second parent is capable of yielding four springs. In the light of this, ‘chromosomes’ in crop management are capable of enhancing farm outputs if they are well ‘cultured’.

**Mutation operation**

After recombination, some children undergo mutation. Mutation operates by inverting each bit in the solution with small probability, usually from 0 to 10%. The rationale is to provide a small amount of randomness, and to prevent solution from been trapped at a local optimum. In the GA used for this work, one-point mutation was operated on.
Local optimization

According to the characteristics of genetic representation adopted in the hybrid genetic algorithm, this study adopts the simplex method to perform the local exploitation of the optimization problem via genetic algorithm. Individuals generated in the initial population or in the genetic operations are not necessarily feasible. They are obtained via random selection. The genetic representation scheme ensures that all constraints are automatically satisfied. In other words, GA allows the under-listed constraints imposed on crop maximization to be automatically satisfied. In this study, crop yield maximization is subject to soil fertility and soil fertility is associated with other factors such as water use, climatic influences, pests and diseases, weed competition, waste, varietal choices of crops and land use. Hence these are constraints imposed on the crop yield maximization.

The objective function is hereby stated again and the solution of the model formulated presented hereafter:

Maximize
\[
\sum_{i} SOF + \sum_{i} WTU + \sum_{i} CLI + \sum_{i} PAD + \sum_{i} WEC + \\
\sum_{i} WST + \sum_{i} VAC + \sum_{i} LAU + \mu
\]

The estimated regression model via linear programming is:

\[
9.2889 = 0.00 + 0.42SOF + 0.00WTU + 0.00CLI + 0.35PAD + \\
0.33WEC + 1.64WST + 0.13VAC + 0.13LAU + \mu
\]

The coefficients of the variables in the model indicates the rate at which the crop yields increase with respect to any change in any of its components which include soil fertility, water use, climatic influence, pests and diseases, weed competition, waste and varietal choice of crop. Specifically, the plus sign implies that all the constraints have positive impact on crop yield maximization. A unit change in soil fertility will result into 0.42 unit change in the crop yield. The result reflects that the impact of water use and climatic influences on crop yield maximization is constant. As regards pests and diseases, a unit change in it will result into 0.33 unit change in the crop yield. This implies that not all pests are destructive. Some of them improve crop yields. A unit change in the weed competition will result into 0.33 unit change in the crop yield. In the same vein, a unit change in the waste will result into 1.64 unit change in the crop yield while a unit change in the varietal choices of crops and land use will yield 0.33 unit change in the crop yields. In a nutshell, the model estimated above indicates that 3 units/tonnes change in all the constraints will give a maximum crop yield of 9 tonnes.

RESULTS

Algorithm testing

The proposed HGA coded in VC++ was tested using the randomly generated example with 10 iterative sites of the components needed. The computational experiments were undertaking on a Pentium IV PC with 128M of memory.

The maximum yield and maximum fertility were randomly generated in set {100, 110, 120, ..., 480, 490, 500}. GA parameters were simulated and set as follows: Soil fertility = 100, Maximum number of crop yields = 200, Crossover rate = 0.8, Mutation rate = 0.1, Penalty factor M = 0.5, random number of Monte Carlo simulation is 1000. All other parameter in the model were set as: a = 0.5, b = 0.1/unit, fw = 6000, uw = 100/unit, MAX_YEILD = 3000units, fr = 2000, ur = 2000, MAX_FERTILITY = 1000units, wr = 1000, us = 500/unit, uc = 500/unit, ua = 500/unit.

Through the 20 experimental trials, in this study, we generated 20 solutions 1758 seconds of CPU time as shown in Figure 2. The trial result yield the average total production of 1,315,945.00, while the total yield range from the highest total yield of 1,324,541.13 to the lowest total yield of 1,308,954.00. Figure 2 shows the best fitness values at each generation as a function of the number of generation.

The graph in Figure 2 indicates the relationship between the trial runs of chromosomes and total farm production. The vertical axis represents the possible farm output generated by the chromosomes on the horizontal axis. Of course, the crop chromosomes are to certain amounts of constraints-soil fertility, climatic influence, etc results into fluctuation in the crop yields obtained from the farmland.

To reveal the effect of the key parameters (that is, soil fertility and maximum number of crop yields) of the proposed HGA on model solution, the study experiments the model with different set of parameters. The sensitivity analysis of crossover rate and mutation rate shows that if crossover rate is higher than zero, it does not affect the model solution significantly; if mutation rate is near 0.1, the solution is better than others. In addition, HGA 20 with combination of soil fertility components and the maximum number of crop yields generates either identical or nearly-identical results.

Here, the study quantifies soil fertility via the LP software considering all the factors that influence it. In the same vein, the corresponding crop yields are indicated too. Table 1 theoretically shows the amount of seedlings (unit) that can be planted for specific durations and the possible yields if the soil fertility is maintained. Different crops take different days to mature. However, maximum crop yields are possible if the soil fertility is at its best.

In the final analysis, it is observed in practice that of crop yield maximization is often influenced by the interactions between different biophysical (most especially soil fertility) and technical components in which most of them are unpredictable and random in nature. Driven by this viewpoint, this study develops a linear mixed-integer programming model to design a genetic algorithm coping with challenging nature of soil fertility. Considering the complexity inherent in a linear mixed-integer programming framework, the study proposed a HGA that was designed to find a maximum crop yield involving uncertain variables such as soil fertility, waste, availability of water, to mention but a few. A series of computational experiments verified that the proposed HGA was efficient in obtaining a near-optimal solution for the crop yield system problem belonging to a class of LP-hard problems. The model shows that 3 units change in
all the constraints will yield farm output of about 9 tonnes to the farmer.

However, it must be noted that despite the proven merits of the proposed formulation and genetic algorithm, further research work needs to be done especially in the area of climate variation and its impact on crop yields.

**CONCLUSION**

One of the major findings of this study is that it is possible to have maximum yields resulting from controlled and scientifically managed interactions of different biophysical and technical components responsible to sustain soil fertility without depleting the soil nutrient. The developed model with the use of genetic algorithm approach helps to quantify various factors that influence soil fertility and relating them to the expected crop yields. The study identifies Soil depth, Availability of Water, Drainage System, Aeration, pH, Mineral Compositions, Organic Matter and Soil Organisms as factors influencing soil fertility. Also, it is observed that every factor identified has expected amount of crop yield associated with it. The regression result shows that for every hundred percent change in soil fertility, holding other factors constant, the crop yield will change by 42%. It is pertinent to know that

---

**Figure 2.** Summary of 20 trial runs.

**Table 1.** Sensitivity changes of soil fertility and maximum number crop yield.

<table>
<thead>
<tr>
<th>Soil fertility</th>
<th>Max yield 100</th>
<th>Max yield 200</th>
<th>Max yield 300</th>
<th>Max yield 400</th>
<th>Max yield 500</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unit</td>
<td>Days</td>
<td>Unit</td>
<td>Days</td>
<td>Unit</td>
</tr>
<tr>
<td>100</td>
<td>13146</td>
<td>45</td>
<td>13106</td>
<td>90</td>
<td>13097</td>
</tr>
<tr>
<td>200</td>
<td>13118</td>
<td>96</td>
<td>13075</td>
<td>226</td>
<td>13059</td>
</tr>
<tr>
<td>300</td>
<td>13140</td>
<td>156</td>
<td>13075</td>
<td>321</td>
<td>13025</td>
</tr>
<tr>
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<td>13086</td>
<td>206</td>
<td>13042</td>
<td>425</td>
<td>13050</td>
</tr>
<tr>
<td>500</td>
<td>13078</td>
<td>275</td>
<td>13059</td>
<td>593</td>
<td>13033</td>
</tr>
</tbody>
</table>
crop yield can be maximized through proper sustenance of soil fertility. This study shows that when soil fertility and other farm related constraints are controlled, farmers can determine the magnitude of their farm outputs.

The relevance of this study cuts across every aspect of agriculture. This means that any problem in agriculture relating to maximization of farm output, minimization of farm expenses and other influencing factors in agriculture can be addressed by genetic algorithm approach.

RECOMMENDATIONS

It is recommended that in order to avoid under or over-utilization of resources, conscious efforts should be made by agricultural extension experts to ensure that farmers imbibe the culture of scientifically quantifying all factors (bio-physical, technical and managerial) required for maximum yield.

From the model developed, it is estimated that the influence of certain factors such as water use on the crop yield is constant while other constraints experience variations. The model shows that 3 units change in all the constraints will yield farm output of about 9 tonnes to the farmer. Hence it is suggested that farmers should regulate those constraints mentioned in this study with respect to the magnitude of their expected farm output. Proper regulation of these constraints will help the farmer to determine the output of his farm land even before the crops are matured.

REFERENCES


APPENDIX

MATLAB program

clear all
f = [-3; -5; -2; -4; -1; -7; -2; -9];
A = [ 2 0 2 3 5 0 5 0
     3 4 0 5 6 6 0 0
     1 2 3 4 0 4 2 4
     8 3 0 6 5 1 0 0
     4 5 4 0 1 3 0 4
     3 7 4 6 7 7 0 7
     6 0 0 6 0 12 0 0
     0 0 7 0 0 3 0 15 ];
b = [10 15 6 12 4 24 4 3];
lb = [0 0 0 0 0 0 0 0];
options = optimset ('LargeScale', 'off', 'Simplex', 'on');
[x,fval] = linprog(f,A,b,[],lb);
z = -fval %Multiplied by -1

Results
z =
 9.2889
x =
 0.0000
 0.4222
 0.0000
 0.0000
 0.3556
 0.3333
 1.6444
 0.1333

Note that objective function should be converted to a minimization problem before entering as done in line 2 of the code. Finally, solution should be multiplied by -1 to the optimized (maximum) solution as done in last but one line, that is, z = fval * -1

Note that: New LP problem.
Max Z = 3\times 1 + 5\times 2 + 2\times 3 + 4\times 4 + 6\times 5 + 7\times 6 + 2\times 7 + 9\times 8
s.t
2\times 1 + 2\times 3 + 3\times 4 + 5\times 5 + 5\times 7 \leq 10
3\times 1 + 4\times 2 + 5\times 4 + 6\times 5 + 6\times 6 \leq 15
.
.
.
7\times 3 + 3\times 6 + 15\times 8 \leq 3
AX \leq b
Hints
Optimization Toolbox

**linprog**

Solve a linear programming problem

\[
\begin{align*}
\min_x & \quad f^T x \\
\text{subject to} & \quad A \cdot x \leq b \\
& \quad A_{eq} \cdot x = b_{eq} \\
& \quad lb \leq x \leq ub
\end{align*}
\]

where \( f, x, b, beq, lb, \) and \( ub \) are vectors and \( A \) and \( A_{eq} \) are matrices.

**Syntax**

\[
x = \text{linprog}(f,A,b,A_{eq},beq)
x = \text{linprog}(f,A,b,A_{eq},beq,lb,ub)
x = \text{linprog}(f,A,b,A_{eq},beq,lb,ub,x0)
x = \text{linprog}(f,A,b,A_{eq},beq,lb,ub,x0,options)
\]

\[
[x,fval] = \text{linprog}(...)
[x,fval,exitflag] = \text{linprog}(...)
[x,fval,exitflag,output] = \text{linprog}(...)
[x,fval,exitflag,output,lambda] = \text{linprog}(...)
\]

**Description**

`linprog` solves linear programming problems.

\( x = \text{linprog}(f,A,b) \) solves \( \min f^T x \) such that \( A \cdot x \leq b \).

\( x = \text{linprog}(f,A,b,A_{eq},beq) \) solves the problem above while additionally satisfying the equality constraints \( A_{eq} \cdot x = beq \). Set \( A=\{} \) and \( b=\{} \) if no inequalities exist.

\( x = \text{linprog}(f,A,b,A_{eq},beq,lb,ub) \) defines a set of lower and upper bounds on the design variables, \( x \), so that the solution is always in the range \( lb \leq x \leq ub \). Set \( A_{eq}=\{} \) and \( beq=\{} \) if no equalities exist.

\( x = \text{linprog}(f,A,b,A_{eq},beq,lb,ub,x0) \) sets the starting point to \( x0 \). This option is only available with the medium-scale algorithm (the LargeScale option is set to 'off' using `optimset`). The default large-scale algorithm and the simplex algorithm ignore any starting point.

\( x = \text{linprog}(f,A,b,A_{eq},beq,lb,ub,x0,options) \) minimizes with the optimization options specified in the structure `options`. Use `optimset` to set these options.

\( [x,fval] = \text{linprog}(...) \) returns the value of the objective function `fun` at the solution \( x \): `fval = f^T x`.

\( [x,lambda,exitflag] = \text{linprog}(...) \) returns a value `exitflag` that describes the exit condition.

\( [x,lambda,exitflag,output] = \text{linprog}(...) \) returns a structure `output` that contains information about the optimization.