Physical Sciences Research International
Vol. 1(3), pp. 54-61, August 2013
Full Length Research Paper

Implementation of Aunu permutation patterns using computer algebra system: A computational approach

A. Mustafa* and A. A. Ibrahim

Department of Mathematics, Usmanu Danfodiyo University, Sokoto, Nigeria.

Accepted 16 April, 2013

ABSTRACT

Aunu permutation patterns are a class of (123) and (132) avoiding permutation patterns with special properties associated with succession and association schemes. These patterns have been used extensively in formulation of different mathematical structures such as groups, graphs and in a study of different algebraic schemes such as cyclic designs and circuit designs. This paper developed an implementation of the (123)-avoiding class using a computer algebra systems (MAPLE), thereby achieving a computational technique for generating the patterns involving larger prime numbers. This povides an improvement of the analytical techniques which have been used before now and which are cumbersome and have already manifested a number of limitations and shortcomings.

Keywords: Permutation, Aunu number, patterns.

*Corresponding author. E-mail: mustaamin_2008@yahoo.com, aminualhaji@yahoo.co.uk.

INTRODUCTION

Aunu permutation patterns first arose, out of attempts to provide some combinatorial interpretations of some succession schemes. In this regard, the patterns were formulated as pairs of successive objects.

$$\lambda_i \lambda_j \text{ such that } i\Theta j \text{ , } i,j \in C_n \quad (1)$$

where Θ is regarded as a precedence parameter associating the *ith and the* jth elements and C_n represents a cyclic structure from which elements are bound to move as permutations. This aspect is well treated in Ibrahim (2005, 2007). Later, it was used as a permutation pattern with some relationships in groups and graphs as in Ibrahim and Audu (2005), where patterns were identified for sample sizes of cardinality larger than five as in Table 1.

Various applications of the patterns were then identified in different aspects of algebra and theoretical computer science. For instance, in Ibrahim (2008), transformations were undertaken of the (132)-avoiding class of these patterns using a binary coding and algorithms to establish some applications in cellular algebra; while in Ibrahim and Audu (2008) studies were concentrated on the applications of the (123)-avoiding class of the patterns in circuit designs and analysis. Recently, some further applications were identified using thin cyclic designs as in Usman and Ibrahim (2011), Automata theory (Ibrahim and Abor, 2011), lattices (Ibrahim and Ibrahim, 2011) and association scheme (Mgami and Ibrahim, 2011).

In an attempt to make this paper more self contained, some basic terms and notions used are enumerated and explained as in the following.

Permutation patterns

An arrangement of the objects $1, 2, ..., n$ is a sequence consisting of these objects arranged in any order. When in addition, a particular order of arrangement is desired, such an arrangement becomes an ordered arrangement governed by a pattern and each such permutation $\sigma \in Sn\{n\}$ naturally results into a certain arrangement of $1, 2, ..., n$ given by:

$$\sigma(1)\sigma(2)...\sigma(n) \quad (2)$$

which is called the arrangement associated with a

Table 1. Aunu Permutation patterns for sample size 5 to 17.

Length of number (n)	(132) avoiding patterns	(123) avoiding patterns
5	1,2,3,4,5	1,4,2,5,3; 1,5,4,3,2;
7	1,2,3,4,5,6,7	1,5,2,6,3,7,4; 1,6,4,2,7,5,3; 1,7,6,5,4,3,2;
11	1,2,3,4,5,6,7,8,9,10,11	1,7,2,8,3,9,4,10,5,11,6; 1,8,4,11,7,3,10,6,2,9,5; 1,9,6,3,11,8,5,2,10,7,4; 1,10,8,6,4,2,11,9,7,5,3; 1,11,10,9,8,7,6,5,4,3,2;
13	1,2,3,4,5,6,7,8,9,10,11,12,13	1,8,2,9,3,10,4,11,5,12,6,13,7; 1,9,4,12,7,2,10,5,13,8,3,11,6; 1,10,6,2,11,7,3,12,8,4,13,9,5; 1,12,10,8,6,4,2,13,11,9,7,5,3; 1,13,12,11,10,9,8,7,6,5,4,3,2;
17	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17	1,10,2,11,3,12,4,13,5,14,6,15,7,16,8,17,9; 1,11,4,14,7,17,10,3,13,6,16,9,2,12,5,15,8; 1,12,6,17,11,5,16,10,4,15,9,3,14,8,2,13,7; 1,13,8,3,15,10,5,17,12,7,2,14,9,4,16,11,6; 1,14,10,6,2,15,11,7,3,16,12,8,4,17,13,9,5; 1,15,12,9,6,3,17,14,11,8,5,2,16,13,10,7,4; 1,16,14,12,10,8,6,4,2,17,15,13,11,9,7,5,3; 1,17,16,15,14,13,12,11,10,9,8,7,6,5,4,3,2;

permutation pattern of points of a nonempty set

$$\Omega = \{1, 2, ..., n\} \quad (3)$$

Permutation avoidance

Given a sequence π consisting of n elements arranged in a given pattern and another sequence σ having m elements such that $m > n$ then σ is said to be contained as a pattern in π provided π has a subsequence which is order isomorphic to σ. If π does not contain σ it is said to avoid it. The set of all the σ-avoiding permutations is denoted $S_n(\sigma)$.

It is sometimes useful to differentiate between subsequences and subwords in permutation avoidance. For instance, if $\sigma = 4132 \in S_4$ then $\pi = 78364521 \in S_8$ contains σ as a subword, since $\rho(8364) = 4132$. However, $\pi = 54321 \in S_s$ does not contain as a subword although it does contain it as a subsequence. Occurrences of subwords can be overlapped. As an example, the sequence $5716243 \in S_7$ contains two occurrences of $\sigma : 7162$ and 6243.

Enumeration procedure

There exist a variety of enumeration systems in counting theory. These include:

(a) Motzkin numbers: $1, 1, 2, 4, 9, 21, 51, ...$ (Chen et al., 2003).

They are used, among others, to enumerate the number of paths along a polygon line from $(0,0)$ to $(n,0)$ and not below the line $y = 0$.

The recursion relation for Motzkin path can be expressed as:

$$U_n = \frac{1}{n}\binom{2n-2}{n-1} \quad (4)$$

where n represents the number of sides in a polygon while U_n is the number of triangles.

(b) Catalan numbers: $1, 1, 2, 5, 14, 42, 132, ...$ (Rechnitzer and Resburg, 2002).

They are used to count the number of triangles in an n-sided polygon. The recursion relation for Catalan numbers is:

$$C_n = \frac{1}{n+1}\binom{2n}{n} \tag{5}$$

where n represents the length of the path while C_n is the possible number of paths.

The n^{th} Catalan number also counts the number of permutations in S_n avoiding σ for all $\sigma \in S_3$

Similarly, the Aunu numbers that avoid (123) as permutation are enumerated as in Ibrahim (2007) by:

$$N(A_n(123)) = \frac{p_n - 1}{2} \tag{6}$$

where $N(A_n(123))$ is the number of all (123)-avoiding patterns while P_n is the n^{th} prime number.

On the other hand, the Aunu numbers that avoids (132) as permutation is enumerated by:

$$A_{n,subwords} = n + (m-1) \tag{7}$$

Where $m < n$ represents the size of subwords that avoid (132) as permutations.

Basic procedure

In what follows, a procedure is provided using Maple (14) generating Aunu patterns for primes. The basic scheme involves identification of even permutations of elements of a non-empty prime. Then, using integer modulo arithmetic's, the cycle were generated given the rules of association between consecutive elements. Lastly, those among the permutation which avoid (123) as permutation patterns, are identified and enumerated.

The following Maple procedure can be used to generate Aunu patterns for primes P_n ³ 5

```
AUNU := proc(p :: posint)
  option remember
  local i, n, sequence;
sequence := Matrix(i, [seq([seq(mod(1 + (i)n , p), n = 0 .. p − 1)], i
     = 1 ..p − 1)]);
end proc;
```

Table 2. Permutation results for N = 5.

1	2	3	4	0
1	3	0	2	4
1	4	2	0	3
1	0	4	3	2
1	2	3	4	0
1	3	0	2	4
1	4	2	0	3
1	0	4	3	2

Table 3. Permutation results for N = 7.

1	2	3	4	5	6	0
1	3	5	0	2	4	6
1	4	0	3	6	2	5
1	5	2	6	3	0	4
1	6	4	2	0	5	3
1	0	6	5	4	3	2

RESULTS

Tables 2 to 10 display the results obtained using the established computational scheme.

CONCLUSION

This paper provides a computational approach to the system of generating Aunu permutation pattern. The paper had not only eased the generation process of these sequences of the positive integers but has also

Table 4. Permutation results for N = 11.

1	2	3	4	5	6	7	8	9	10	0
1	3	5	7	9	0	2	4	6	8	10
1	4	7	10	2	5	8	0	3	6	9
1	5	9	2	6	10	3	7	0	4	8
1	6	0	5	10	4	9	3	8	2	7
1	7	2	8	3	9	4	10	5	0	6
1	8	4	0	7	3	10	6	2	9	5
1	9	6	3	0	8	5	2	10	7	4
1	10	8	6	4	2	0	9	7	5	3
1	0	10	9	8	7	6	5	4	3	2

Table 5. Permutation results for N = 13.

1	2	3	4	5	6	7	8	9	10	11	12	0
1	3	5	7	9	11	0	2	4	6	8	10	12
1	4	7	10	0	3	6	9	12	2	5	8	11
1	5	9	0	4	8	12	3	7	11	2	6	10
1	6	11	3	8	0	5	10	2	7	12	4	9
1	7	0	6	12	5	11	4	10	3	9	2	8
1	8	2	9	3	10	4	11	5	12	6	0	7
1	9	4	12	7	2	10	5	0	8	3	11	6
1	10	6	2	11	7	3	12	8	4	0	9	5
1	11	8	5	2	12	9	6	3	0	10	7	4
1	12	10	8	6	4	2	0	11	9	7	5	3
1	0	12	11	10	9	8	7	6	5	4	3	2

Table 6. Permutation results for N = 17.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	0
1	3	5	7	9	11	13	15	0	2	4	6	8	10	12	14	16
1	4	7	10	13	16	2	5	8	11	14	0	3	6	9	12	15
1	5	9	13	0	4	8	12	16	3	7	11	15	2	6	10	14
1	6	11	16	4	9	14	2	7	12	0	5	10	15	3	8	13
1	7	13	2	8	14	3	9	15	4	10	16	5	11	0	6	12
1	8	15	5	12	2	9	16	6	13	3	10	0	7	14	4	11
1	9	0	8	16	7	15	6	14	5	13	4	12	3	11	2	10
1	10	2	11	3	12	4	13	5	14	6	15	7	16	8	0	9
1	11	4	14	7	0	10	3	13	6	16	9	2	12	5	15	8
1	12	6	0	11	5	16	10	4	15	9	3	14	8	2	13	7
1	13	8	3	15	10	5	0	12	7	2	14	9	4	16	11	6
1	14	10	6	2	15	11	7	3	16	12	8	4	0	13	9	5
1	15	12	9	6	3	0	14	11	8	5	2	16	13	10	7	4
1	16	14	12	10	8	6	4	2	0	15	13	11	9	7	5	3
1	0	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2

established a generation of the analytical methods used before now. Further, the current approach has introduced another area of the computation process and analytic methods in algebra.

RECOMMENDATION

The studies proposed in this paper are by no means exhaustive neither are the numerous results arising there

Table 7. Permutation results for N = 19.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	0
1	3	5	7	9	11	13	15	17	0	2	4	6	8	10	12	14	16	18
1	4	7	10	13	16	0	3	6	9	12	15	18	2	5	8	11	14	17
1	5	9	13	17	2	6	10	14	18	3	7	11	15	0	4	8	12	16
1	6	11	16	2	7	12	17	3	8	13	18	4	9	14	0	5	10	15
1	7	13	0	6	12	18	5	11	17	4	10	16	3	9	15	2	8	14
1	8	15	3	10	17	5	12	0	7	14	2	9	16	4	11	18	6	13
1	9	17	6	14	3	11	0	8	16	5	13	2	10	18	7	15	4	12
1	10	0	9	18	8	17	7	16	6	15	5	14	4	13	3	12	2	11
1	11	2	12	3	13	4	14	5	15	6	16	7	17	8	18	9	0	10
1	12	4	15	7	18	10	2	13	5	16	8	0	11	3	14	6	17	9
1	13	6	18	11	4	16	9	2	14	7	0	12	5	17	10	3	15	8
1	14	8	2	15	9	3	16	10	4	17	11	5	18	12	6	0	13	7
1	15	10	5	0	14	9	4	18	13	8	3	17	12	7	2	16	11	6
1	16	12	8	4	0	15	11	7	3	18	14	10	6	2	17	13	9	5
1	17	14	11	8	5	2	18	15	12	9	6	3	0	16	13	10	7	4
1	18	16	14	12	10	8	6	4	2	0	17	15	13	11	9	7	5	3
1	0	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2

Table 8. Permutation results for N = 23.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	0
1	3	5	7	9	11	13	15	17	19	21	0	2	4	6	8	10	12	14	16	18	20	22
1	4	7	10	13	16	19	22	2	5	8	11	14	17	20	0	3	6	9	12	15	18	21
1	5	9	13	17	21	2	6	10	14	18	22	3	7	11	15	19	0	4	8	12	16	20
1	6	11	16	21	3	8	13	18	0	5	10	15	20	2	7	12	17	22	4	9	14	19
1	7	13	19	2	8	14	20	3	9	15	21	4	10	16	22	5	11	17	0	6	12	18
1	8	15	22	6	13	20	4	11	18	2	9	16	0	7	14	21	5	12	19	3	10	17
1	9	17	2	10	18	3	11	19	4	12	20	5	13	21	6	14	22	7	15	0	8	16
1	10	19	5	14	0	9	18	4	13	22	8	17	3	12	21	7	16	2	11	20	6	15
1	11	21	8	18	5	15	2	12	22	9	19	6	16	3	13	0	10	20	7	17	4	14
1	12	0	11	22	10	21	9	20	8	19	7	18	6	17	5	16	4	15	3	14	2	13
1	13	2	14	3	15	4	16	5	17	6	18	7	19	8	20	9	21	10	22	11	0	12
1	14	4	17	7	20	10	0	13	3	16	6	19	9	22	12	2	15	5	18	8	21	11
1	15	6	20	11	2	16	7	21	12	3	17	8	22	13	4	18	9	0	14	5	19	10
1	16	8	0	15	7	22	14	6	21	13	5	20	12	4	19	11	3	18	10	2	17	9
1	17	10	3	19	12	5	21	14	7	0	16	9	2	18	11	4	20	13	6	22	15	8
1	18	12	6	0	17	11	5	22	16	10	4	21	15	9	3	20	14	8	2	19	13	7

Table 8. Cont.

1	19	14	9	4	22	17	12	7	2	20	15	10	5	0	18	13	8	3	21	16	11	6
1	20	16	12	8	4	0	19	15	11	7	3	22	18	14	10	6	2	21	17	13	9	5
1	21	18	15	12	9	6	3	0	20	17	14	11	8	5	2	22	19	16	13	10	7	4
1	22	20	18	16	14	12	10	8	6	4	2	0	21	19	17	15	13	11	9	7	5	3

Table 9. Permutation results for N = 29.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	0
1	3	5	7	9	11	13	15	17	19	21	23	25	27	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28
1	4	7	10	13	16	19	22	25	28	2	5	8	11	14	17	20	23	26	0	3	6	9	12	15	18	21	24	27
1	5	9	13	17	21	25	0	4	8	12	16	20	24	28	3	7	11	15	19	23	27	2	6	10	14	18	22	26
1	6	11	16	21	26	2	7	12	17	22	27	3	8	13	18	23	28	4	9	14	19	24	0	5	10	15	20	25
1	7	13	19	25	2	8	14	20	26	3	9	15	21	27	4	10	16	22	28	5	11	17	23	0	6	12	18	24
1	8	15	22	0	7	14	21	28	6	13	20	27	5	12	19	26	4	11	18	25	3	10	17	24	2	9	16	23
1	9	17	25	4	12	20	28	7	15	23	2	10	18	26	5	13	21	0	8	16	24	3	11	19	27	6	14	22
1	10	19	28	8	17	26	6	15	24	4	13	22	2	11	20	0	9	18	27	7	16	25	5	14	23	3	12	21
1	11	21	2	12	22	3	13	23	4	14	24	5	15	25	6	16	26	7	17	27	8	18	28	9	19	0	10	20
1	12	23	5	16	27	9	20	2	13	24	6	17	28	10	21	3	14	25	7	18	0	11	22	4	15	26	8	19
1	13	25	8	20	3	15	27	10	22	5	17	0	12	24	7	19	2	14	26	9	21	4	16	28	11	23	6	18
1	14	27	11	24	8	21	5	18	2	15	28	12	25	9	22	6	19	3	16	0	13	26	10	23	7	20	4	17
1	15	0	14	28	13	27	12	26	11	25	10	24	9	23	8	22	7	21	6	20	5	19	4	18	3	17	2	16
1	16	2	17	3	18	4	19	5	20	6	21	7	22	8	23	9	24	10	25	11	26	12	27	13	28	14	0	15
1	17	4	20	7	23	10	26	13	0	16	3	19	6	22	9	25	12	28	15	2	18	5	21	8	24	11	27	14
1	18	6	23	11	28	16	4	21	9	26	14	2	19	7	24	12	0	17	5	22	10	27	15	3	20	8	25	13
1	19	8	26	15	4	22	11	0	18	7	25	14	3	21	10	28	17	6	24	13	2	20	9	27	16	5	23	12
1	20	10	0	19	9	28	18	8	27	17	7	26	16	6	25	15	5	24	14	4	23	13	3	22	12	2	21	11
1	21	12	3	23	14	5	25	16	7	27	18	9	0	20	11	2	22	13	4	24	15	6	26	17	8	28	19	10
1	22	14	6	27	19	11	3	24	16	8	0	21	13	5	26	18	10	2	23	15	7	28	20	12	4	25	17	9
1	23	16	9	2	24	17	10	3	25	18	11	4	26	19	12	5	27	20	13	6	28	21	14	7	0	22	15	8
1	24	18	12	6	0	23	17	11	5	28	22	16	10	4	27	21	15	9	3	26	20	14	8	2	25	19	13	7
1	25	20	15	10	5	0	24	19	14	9	4	28	23	18	13	8	3	27	22	17	12	7	2	26	21	16	11	6
1	26	22	18	14	10	6	2	27	23	19	15	11	7	3	28	24	20	16	12	8	4	0	25	21	17	13	9	5
1	27	24	21	18	15	12	9	6	3	0	26	23	20	17	14	11	8	5	2	28	25	22	19	16	13	10	7	4
1	28	26	24	22	20	18	16	14	12	10	8	6	4	2	0	27	25	23	21	19	17	15	13	11	9	7	5	3
1	0	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2

Table 10. Permutation results for N = 31.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	0
1	3	5	7	9	11	13	15	17	19	21	23	25	27	29	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
1	4	7	10	13	16	19	22	25	28	0	3	6	9	12	15	18	21	24	27	30	2	5	8	11	14	17	20	23	26	29
1	5	9	13	17	21	25	29	2	6	10	14	18	22	26	30	3	7	11	15	19	23	27	0	4	8	12	16	20	24	28
1	6	11	16	21	26	0	5	10	15	20	25	30	4	9	14	19	24	29	3	8	13	18	23	28	2	7	12	17	22	27
1	7	13	19	25	0	6	12	18	24	30	5	11	17	23	29	4	10	16	22	28	3	9	15	21	27	2	8	14	20	26
1	8	15	22	29	5	12	19	26	2	9	16	23	30	6	13	20	27	3	10	17	24	0	7	14	21	28	4	11	18	25
1	9	17	25	2	10	18	26	3	11	19	27	4	12	20	28	5	13	21	29	6	14	22	30	7	15	23	0	8	16	24
1	10	19	28	6	15	24	2	11	20	29	7	16	25	3	12	21	30	8	17	26	4	13	22	0	9	18	27	5	14	23
1	11	21	0	10	20	30	9	19	29	8	18	28	7	17	27	6	16	26	5	15	25	4	14	24	3	13	23	2	12	22
1	12	23	3	14	25	5	16	27	7	18	29	9	20	0	11	22	2	13	24	4	15	26	6	17	28	8	19	30	10	21
1	13	25	6	18	30	11	23	4	16	28	9	21	2	14	26	7	19	0	12	24	5	17	29	10	22	3	15	27	8	20
1	14	27	9	22	4	17	30	12	25	7	20	2	15	28	10	23	5	18	0	13	26	8	21	3	16	29	11	24	6	19
1	15	29	12	26	9	23	6	20	3	17	0	14	28	11	25	8	22	5	19	2	16	30	13	27	10	24	7	21	4	18
1	16	0	15	30	14	29	13	28	12	27	11	26	10	25	9	24	8	23	7	22	6	21	5	20	4	19	3	18	2	17
1	17	2	18	3	19	4	20	5	21	6	22	7	23	8	24	9	25	10	26	11	27	12	28	13	29	14	30	15	0	16
1	18	4	21	7	24	10	27	13	30	16	2	19	5	22	8	25	11	28	14	0	17	3	20	6	23	9	26	12	29	15
1	19	6	24	11	29	16	3	21	8	26	13	0	18	5	23	10	28	15	2	20	7	25	12	30	17	4	22	9	27	14
1	20	8	27	15	3	22	10	29	17	5	24	12	0	19	7	26	14	2	21	9	28	16	4	23	11	30	18	6	25	13
1	21	10	30	19	8	28	17	6	26	15	4	24	13	2	22	11	0	20	9	29	18	7	27	16	5	25	14	3	23	12
1	22	12	2	23	13	3	24	14	4	25	15	5	26	16	6	27	17	7	28	18	8	29	19	9	30	20	10	0	21	11
1	23	14	5	27	18	9	0	22	13	4	26	17	8	30	21	12	3	25	16	7	29	20	11	2	24	15	6	28	19	10
1	24	16	8	0	23	15	7	30	22	14	6	29	21	13	5	28	20	12	4	27	19	11	3	26	18	10	2	25	17	9
1	25	18	11	4	28	21	14	7	0	24	17	10	3	27	20	13	6	30	23	16	9	2	26	19	12	5	29	22	15	8
1	26	20	14	8	2	27	21	15	9	3	28	22	16	10	4	29	23	17	11	5	30	24	18	12	6	0	25	19	13	7
1	27	22	17	12	7	2	28	23	18	13	8	3	29	24	19	14	9	4	30	25	20	15	10	5	0	26	21	16	11	6
1	28	24	20	16	12	8	4	0	27	23	19	15	11	7	3	30	26	22	18	14	10	6	2	29	25	21	17	13	9	5
1	29	26	23	20	17	14	11	8	5	2	30	27	24	21	18	15	12	9	6	3	0	28	25	22	19	16	13	10	7	4
1	30	28	26	24	22	20	18	16	14	12	10	8	6	4	2	0	29	27	25	23	21	19	17	15	13	11	9	7	5	3
1	0	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2

from. It is therefore recommended that further researches be conducted to develop a computational procedure for restricted permutation of these categories, such as the (123)-avoiding and (132)-avoiding patterns of Aunu permutations addition researchers should exploit more properties of the resulting patterns especially in terms of matrix algebra, vector algebra and polynomials.

REFERENCES

Chen YCW, Deng Y, Laura LM, 2003. Motzkin Paths and reduced Decompositions for Permutations with Forbidden Patterns. The Elect J Comb, 9(2):1-13.

Ibrahim AA, 2005. On the combinatorics of succession using a 5-element sample. Abacus, Journal of Mathematics Association of Nigeria, 32(2B):410-415.

Ibrahim AA, 2007. A counting scheme and some algebraic properties of a class of special permutation patterns. Journal of Discrete Mathematical Sciences and Cryptography, 10(4):537-546.

Ibrahim AA, 2008. Some transformation schemes involving the special (132) – avoiding permutation patterns and a binary coding: An algorithmic approach. Asian J Algebra, 1(1):10-14.

Ibrahim AA, Abor GI, 2011. On the use of automata models in the study of a class of (123)-avoiding permutation patterns: Applications in circuit design and in number theory. J Sci Technol Res, 10(1):71-77.

Ibrahim AA, Audu MS, 2005. Some Group Theoretic Properties of Certain Class of (123) and (132)-Avoiding patterns numbers: An enumeration scheme. Afr J Nat Sci, 8:79-84.

Ibrahim AA, Audu MS, 2008. On Stable Variety of Cayley graphs for Efficient Interconnection Networks. Proceedings of Annual National Conference of Mathematical Association of Nigeria (MAN) held at Federal College of Education Technical, Gusau 26th– 30th August, 2008. pp: 156-161.

Ibrahim M, Ibrahim AA, 2011. Algebraic Theoretic Properties of the (132)-Avoiding Class of Aunu Permutation Patterns: Applications in Lattices. Proceedings of Annual Conference of IRDI Research and Development Network, 6(2):108-111.

Mgami MS, Ibrahim AA, 2011. Construction of Association Scheme Using Some (123)-avoiding Class of Aunu Patterns. Nig J Basic Appl Sci, 19(1):5-8.

Rechnitzer A, Resburg EJJ, 2002. Exchange Symmetries in Motzkin Path and Bar graph Models of Copolymer Adsorption. The Elect J Comb, 9:1-13.

Usman A, Ibrahim AA, 2011. Enumeration Scheme of (123)-Avoiding To Aunu Groups Using Thin Cyclic Design. Proceedings of Annual National Conference of Mathematical Association of Nigeria (MAN) held at Federal College Polytechnic Ado-Ekiti 4th– 9th September, 2011. pp. 99-108.