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The nth moment of X^cY^d

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ABSTRACT

We propose an alternative method of obtaining the nth moment of the joint distribution of the cth power of the random variable X and the dth power of the random variable Y about zero for all non-negative values of c and d. The method, denoted by μ_n (c, d), may be termed 'moment of power generating function (mpgf)'. It exists for all continuous distributions unlike in contenders-the factorial moments and the moment generating functions which do not always exist. The proposed μ_n (c, d) is illustrated with some continuous bivariate distributions and is shown to be easy to use even when the powers of the random variables being considered are non-negative real numbers that need not be integers. The results obtained using μ_n (c, d) are the same as results obtained using other methods such as moment generating functions when they exist.

Keywords: Moment generating function, joint distribution, integers, probability density function, non-negative real numbers, Skewness, Kurtosis, marginal distribution.

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INTRODUCTION

The expected value $E\left(X^{cn}Y^{dn}\right)$ is usually interpreted as the $(cn, dn)th$ moment of the joint distribution of the random variables X and Y about zero. However, if written as $E\left(X^cY^d\right)^n$, then $E\left(X^{cn}Y^{dn}\right)$ may be viewed as the nth moment of the joint distribution of X^c *and* Y^d about zero where n is a non-negative interger while c and d are non-negative real numbers but not necessarily intergers. This situation could arise for example when a researcher has preliminary information on the various X and Y but his primary interest is to obtain information on the joint distribution and characteristics of some non-linear functions of these variables (Uche, 2003; Oyeka, 1996). The moment $E\left(X^cY^d\right)^n$ may simply be obtained using the usual definition of expected values of random variables or by using relative factorial moments or moment generating functions (Baisnab and Manoranjan, 1993). The problem with the latter two is that they do not exist for every probability distribution and even if they exist, their use may be rather difficult especially in finding higher moments when repeated differentiations and evaluations may be involved.

In this paper, we propose a method that is easier to use based on methods applied by some authors (Oyeka et al., 2009; Spiegel, 1998; Freund, 1992); specifically, we intend to develop an alternative method of obtaining the nth moment or expected value of the joint distribution of the cth power of the random variable X and the dth power of the random variable Y about zero; where n is a non-negative integer while c and d are non-negative real numbers. We here assume according to Freund (1992) and Hay (1973) that both X and Y are continuously differentiable on the real line or over their range of definition with joint probability density function (PDF) f(x,y). For lack of a better notation, we here use $\mu_n(c,d)$ to denote the nth moment or expected value of the joint distribution of the cth power of the random variable X and the dth power of the random variable Y about zero. Also from its definition, $\mu_n(c,d)$ may be viewed as a generating function of the moments of the joint distribution of powers of the random variables X and Y about zero for various values of n. Thus, $\mu_n(c,d)$ is a sort of 'moment of power generating function (mpgf)" of the joint distribution of powers of X and Y, and may be termed. Clearly, given the joint distribution of X and Y, any functions of X and Y such as $X^c and\ Y^d$ also have their own joint distributions which can easily be found. Strictly speaking, in finding $E\left(X^cY^d\right)^n$, one would need to first find and then use the joint distribution of $X^c and\ Y^d$ in the calculations. However, as illustrated below, the results obtained when using either the joint distribution of $X^c and\ Y^d$ or simply the joint distribution of X and Y are always the same. Hence, in finding the moments of the distribution of some functions such as $X^c and\ Y^d$ of given random variables X and Y; it is not necessary to find and use the joint distribution of these functions. It is sufficient to simply use only the joint distribution of the random variables themselves in the calculations. This approach is adopted here.

THE PROPOSED METHOD

Now,

$$\mu_n(c,d) = E\left(X^cY^d\right)^n, (c \geq 0; d \geq 0) \tag{1}$$
$$for\ n = 0,1,2,3,...;$$
$$that\ is$$
$$\begin{aligned}\mu_n(c,d) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(x^c\right)^n\left(y^d\right)^n f(x,y)\,dxdy \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^{cn}y^{dn} f(x,y)\,dxdy \\ &= \mu(cn,dn)\end{aligned}$$

$where\ \mu(cn,dn)\ is\ the\ (cn,dn)th$ moment of the joint distribution of X and Y about zero. The first part of the above equation shows that the nth moment of the joint distribution of $X^c and\ Y^d$ can be found for all non-negative real values of c and d including non integral values. Furthermore, the second part of the above equation shows that this moment can be found as the $(cn,dn)th$ moment of the joint distribution of X and Y about zero. In other words;

$$\mu_n(c,d) = \mu'(cn,dn) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^{cn}y^{dn} f(x,y)\,dxdy \tag{2}$$

Note that $\mu_0(c,d) = 1$ for all $(c \geq 0; d \geq 0)$ as expected. Equation 2 may be used to obtain any desired moment of the joint distribution of $X^c and\ Y^d$, for example: The first moment (n = 1) of the joint distribution of $X^c and\ Y^d$ about zero is,

$$\mu\left(X^cY^d\right) = \mu_1(c,d) \tag{3}$$
$$where\ \mu_1(c,d) = \mu'(c,d) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^c y^d f(x,y)\,dxdy.$$

And the joint variation of the distribution of $X^c and\ Y^d$ is

$$\mathrm{var}\left(X^cY^d\right) = \mu_2(c,d) - \mu_1(c,d)^2 \tag{4}$$

Which is the (2nd) second moment of the joint distribution of X^cY^d about its mean. The Skewness (Sk) and Kurtosis

(Ku) of this distribution can also be easily calculated using equation 2 if desired. Thus, the Skewness of the joint distribution of X^c *and* Y^d is given as,

$$SK\left(X^cY^d\right)=\frac{\mu_3(c,d)-3\mu_2(c,d)\mu_1(c,d)+2\mu_1(c,d)^3}{\left(\mu_2(c,d)-\mu_1(c,d)^2\right)^{\frac{3}{2}}} \qquad (5)$$

While the corresponding Kurtosis (KU) is obtained as,

$$KU\left(X^cY^d\right)=\frac{\mu_4(c,d)-4\mu_3(c,d)\mu_1(c,d)+6\mu_2(c,d)\mu_1(c,d)^2-3\mu_1(c,d)^4}{\left(\mu_2(c,d)-\mu_1(c,d)^2\right)^2} \qquad (6)$$

If in Equation 2, we set (d = 0), we obtain the nth moment of the marginal distribution of X^c about zero; the nth moment of the marginal distribution of X^c that is,

$$_x\mu_n(c)=\mu_n(c,o) \qquad (7)$$

Similarly the nth moment of the marginal distribution of Y^d about zero; is obtained by setting (c = 0) in Equation 2 that is:

$$_y\mu_n(c)=\mu_n(o,d) \qquad (8)$$

The Skewness (SK) and Kurtosis (KU) of the marginal distribution of X^c *and* Y^d may also be obtain, for example the Skewness of the marginal distribution of X^c is obtained using Equation 7 as:

$$SK\left(X^c\right)=\frac{_x\mu_3(c)-3\,_x\mu_2(c)\,_x\mu_1(c)+2\,_x\mu_1(c)^3}{\left(\mu_{x_2}(c)-\mu_{x_1}(c)^2\right)^{\frac{3}{2}}} \qquad (9)$$

While the corresponding Kurtosis is,

$$KU\left(X^c\right)=\frac{\mu_{x_4}(c)-4\mu_{x_3}(c)\mu_{x_1}(c)+6\mu_{x_2}(c)\mu_{x_1}(c)^2-3\mu_{x_1}(c)^4}{\left(\mu_{x_2}(c)-\mu_{x_1}(c)^2\right)^2} \qquad (10)$$

The Skewness and Kurtosis of the marginal distribution of Y^d are similarly obtained using Equation 8.

Illustrative example

Suppose two continuous random variables X and Y have the joint pdf;

$$f(x,y)=\frac{2}{\beta^2}xy\ell^{-\frac{y}{\beta}},(0<x<1;y>0) \qquad (11)$$

Then we have from Equation 2 that;

$$\mu_n(c,d) = E\left(X^cY^d\right)^n = EX^{cn}Y^{dn}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^{cn}y^{dn} f(x,y)\,dxdy$$

$$= \frac{2}{\beta^2}\int_{-\infty}^{\infty}\int_0^1 x^{cn}y^{dn}xy\ell^{-\frac{y}{\beta}}dxdy,$$

$$(let\ v = \frac{y}{\beta}\ \therefore\ dy = \beta dv)$$

$$\mu_n(c,d) = \frac{2\beta^{dn}\overline{)(dn}+2)}{cn+2} \qquad (12)$$

Note from Equation 12 that here as always $\mu_0(c,d)=1.$ Note also that for n = 1, we have used equations 3 and 9, therefore the mean of the joint distribution of $X^c\,and\,Y^d$ is:

$$\mu_1(c,d) = \frac{2\beta^d\overline{)d}+2}{c+2}.$$

As noted earlier, c and d need not be integers. Thus, if here $c=\frac{1}{3}\ and\ d=\frac{1}{2}$ then the mean of the joint distribution of $X^{\frac{1}{3}}\ and\ Y^{\frac{1}{2}}$ is:

$$\mu\left(X^{\frac{1}{3}}Y^{\frac{1}{2}}\right) = \mu_1\left(\frac{1}{3},\frac{1}{2}\right) = \frac{2\beta^{\frac{1}{2}}\overline{)\frac{1}{2}}+2}{\frac{1}{3}+2} = \frac{2\beta^{\frac{1}{2}}\left(1+\frac{1}{2}\right)\left(\frac{1}{2}\overline{)\pi}\right)}{\frac{7}{3}} = 1.14\sqrt{\beta}\ approx$$

While using equation 4 and 12 we have that the variance of the joint distribution of X^cY^d is:

$$Var\left(X^{\frac{1}{3}}Y^{\frac{1}{2}}\right) = \frac{2\beta^{\frac{1}{2}(2)}\overline{)\frac{1}{2}}(2)+2}{\frac{1}{3}(2)+2} - \left(\frac{9\sqrt{\beta\pi}}{14}\right)^2 = \frac{2\beta^2}{\frac{8}{3}} - \left(\frac{9\sqrt{\beta\pi}}{14}\right)^2 = 0.20\beta$$

Equation 12 would have also been obtained using the joint distribution of X^cY^d given the joint distribution of X and Y. Thus, if we let $let\ U = X^c\ and\ V = Y^d$, then the joint distribution of U and V given the above joint distribution of X and Y in equation 10 is:

$$g(u,v) = \frac{2}{cd_{\beta^2}}U^{\frac{2}{c}-1}\,V^{\frac{2}{d}-1}\ell^{-\frac{V^{\frac{1}{d}}}{\beta}},(0<U<1)(V>0) \qquad (13)$$

Hence

$$EU^nV^n = \frac{2}{cd_{\beta^2}} \int_0^1 \int_0^\infty U^{n+(2/c)-1} V^{n+(2/d)-1} \ell^{-\frac{V^{\frac{1}{d}}}{\beta}} dudv,$$

$$\left(let\ z = \frac{V^{\frac{1}{d}}}{\beta} \therefore V = \beta^d z^d \therefore dv = d\beta^d z^{d-1} dz \right)$$

$$= \int_0^1 \int_0^\infty U^{\frac{n+2}{c}-1} \left(\beta^d z^d\right)^{\frac{n+2}{d}-1} \beta^d z^{d-1} \ell^{-z} dudz$$

That is,

$$\mu_n(c,d) = EU^nV^n = \frac{2\beta^{dn}\overline{)d} + 2}{cn+2} \tag{14}$$

The same result as obtained in Equation 12 based directly on the joint distribution of X and Y. The moment, $\mu_{x_n}(c)$ of the marginal distribution of X^c for this illustrative example may be easily obtained by setting d = 0 in either Equation 12 or 14 or on the basis of the marginal distribution of X^c given in Equation 11 as:

$$\mu_{xn}(c) = \mu_n(c,o) = \frac{2}{cn+2} \tag{15}$$

Similarly

$\mu_{y_n}(d)$, the corresponding nth moment of the marginal distribution of Y^d is:

$$\mu_{yn}(d), = \mu_n(o,d) = \beta^{dn}\overline{)dn} + 2 \tag{16}$$

As an illustrative example the first moment (n = 1) of the marginal distribution of $X^{\frac{1}{3}}$ about zero is obtained using Equation 15 as:

$$\mu_x\left(X^{\frac{1}{3}}\right) = \mu_x^1\left(X^{\frac{1}{3}}\right) = \frac{2}{\frac{1}{3}(1)+2} = \frac{6}{7}$$

While the corresponding variance is obtained from Equations 7 and 15 for $c = \frac{1}{3}$ as:

$$Var\ (x^{\frac{1}{3}}) = \mu_{x_2}\left(\frac{1}{3}\right) - \mu_{x_1}\left(X^{\frac{1}{3}}\right) = \frac{3}{4} - \left(\frac{6}{7}\right)^2 = 0.02$$

The Skewness of the marginal distribution of $x^{\frac{1}{3}}$ is easily obtained from Equation 9 with

$$c = \frac{1}{3} \; as \; \frac{-5.78}{0.02^{\frac{3}{2}}} = -2043.5 \; skewness$$

The kurtosis (KU) may also be found using equation 10 with $c = \frac{1}{3}$ if desired. Similar calculations may be made using equation 16 to obtain desired moments for the marginal distribution of Y^d. Suppose the random variables X and Y have the joint probability density function (pdf)

$$f(x, y) = (x + y)\frac{\ell^{-\left(\frac{x}{\beta_1}+\frac{y}{\beta_2}\right)}}{\beta_1\beta_2\left(\beta_1+\beta_2\right)}, \left(x > 0, y > 0\right) \tag{17}$$

The $\mu_n\left(c, d\right)$ corresponding to this distribution is obtained using Equation 2 as:

$$\mu_n\left(c, d\right) = E\left(X^c Y^d\right)^n = \int_0^1\int_0^1 x^{cn} y^{dn} f\left(x^{cn}, y^{dn}\right) dxdy$$
$$= \frac{1}{\beta_1\beta_2\left(\beta_1+\beta_2\right)}\int_0^1\int_0^1 (x + y)\ell^{-\left(\frac{x}{\beta_1}+\frac{y}{\beta_2}\right)} dxdy$$

$letting \; u = \frac{x}{\beta_1} \; and \; v = \frac{y}{\beta_2}$ and integrating and evaluating yields

$$\mu_n\left(c, d\right) = \frac{\beta_1^{cn}\beta_2^{dn}}{\left(\beta_1+\beta_2\right)}\left(\beta_1\overline{)(cn+2)}\overline{)(dn+1)} + \beta_2\overline{)(dn+2)}\overline{)(cn+1)}\right)$$
$$\mu_n\left(c, d\right) = \frac{\beta_1^{cn}\beta_2^{dn}}{\left(\beta_1+\beta_2\right)}\overline{)(dn+1)}\overline{)(cn+1)}\left(\beta_1\overline{)(cn+1)}\beta_2\overline{)(dn+1)}\right) \tag{18}$$

To find the mean of the joint distribution of X and Y, we set c = d = 1 and n = 1 in Equation 18 to obtain

$$\mu_1(1,1) = \frac{\beta_1\beta_2\left(\beta_1\overline{)1}+2\,\overline{)1}+1+\beta_2\overline{)1}+2\overline{)1}+1\right)}{\beta_1+\beta_2} = \frac{\beta_1\beta_2\left(2\beta_1+2\,\beta_2\right)}{\beta_1+\beta_2} = 2\beta_1\beta_2$$

The variance of this joint distribution is obtain using Equation 4 in 18 as:

$$Var(x, y) = \mu_2(1,1) - \mu_1(1,1)^2 = 12\beta_1^2\beta_2^2 - \left(2\beta_1\beta_2\right)^2 = 8\beta_1^2\beta_2^2$$

The Skewness of this joint distribution is similarly obtained using Equation 5 in 18 with c = d = 1 as:

$$\frac{88\beta_1^3\beta_2^3}{\left(8\beta_1^2\beta_2^2\right)^{\frac{3}{2}}} = 11\overline{)\frac{2}{4}} = 3.9$$

It is easily calculated from equation 18 for $c = \frac{3}{2}$ *and* $d = \frac{1}{2}$ that the first moment of the joint distribution of $X^{\frac{3}{2}}$ *and* $Y^{\frac{1}{2}}$ about zero is:

$$\mu_1\left(\frac{3}{2},\frac{1}{2}\right) = EX^{\frac{3}{2}}Y^{\frac{1}{2}} = \frac{\frac{3\pi}{16}\times\beta_1^{\frac{3}{2}}\beta_2^{\frac{1}{2}}\left(5\beta_1+3\beta_2\right)}{\beta_1+\beta_2}$$

$$and\ the\ second\ moment\ (n=2)\ is,$$

$$\mu_2\left(\frac{3}{2},\frac{1}{2}\right) = \frac{12\beta_1^3\beta_2\left(2\beta_1+\beta_2\right)}{\beta_1+\beta_2}$$

The nth moment of the marginal distributions of X^c *and* Y^d about zero are obtained from Equation 18 respectively as:

$$\mu_{xn}(c,0) = \mu_{cn}(0,c) = \frac{\beta_1^{cn}\left(\beta_1\overline{)(cn}+2)+\beta_2\overline{)(cn}+1)\right)}{\beta_1+\beta_2} = \frac{cn\overline{)cn}\beta_1^{cn}\left((cn+1)\beta_1+\beta_2\right)}{\beta_1+\beta_2} \qquad (19)$$

and

$$\mu_{yn}(d) = \mu_{yn}(o,d) = \frac{\beta_1\beta_2^{dn}}{\left(\beta_1+\beta_2\right)}\left(\overline{)(dn}+1)+\beta_2^{dn+1}\overline{)(dn}+2)\right)$$

$$= \frac{\beta_n^{dn}dn\overline{)(dn)}\left(\beta_1+(dn+1)\beta_2\right)}{\left(\beta_1+\beta_2\right)} \qquad (20)$$

For this example if $c = \frac{1}{3}$, then the first moment (n = 1) of the marginal distribution of $X^{\frac{1}{3}}$ is obtained from Equation 19 for n = 1 and $c = \frac{1}{3}$ as,

$$EX^{\frac{1}{3}} = \mu_{x_1}(\frac{1}{3}) = \frac{\dfrac{\beta_1^{\frac{1}{3}}\overline{)\frac{1}{3}}\left(4\beta_1+3\beta_2\right)}{9}}{\beta_1+\beta_2}$$

And the corresponding second moment (n = 2) is,

$$\mu_{x_2}(\frac{1}{3}) = \frac{\dfrac{2\beta_1^{\frac{2}{3}}\overline{)\frac{2}{3}}\left(5\beta_1+3\beta_2\right)}{9}}{\beta_1+\beta_2}$$

To illustrate the needed modification we note that the marginal distribution of X in Equation 17 is:

$$f(x) = \frac{x\ell^{-\frac{x}{\beta_1}}+\beta_2\ell^{-\frac{x}{\beta_1}}}{\beta_1\left(\beta_1+\beta_2\right)},(x>0) \qquad (21)$$

Therefore the pdf of $Y = X^c\left(c>0\right)$ is

$$g(y) = \frac{y^{\frac{2}{c}-1}\ell^{-\frac{y^{\frac{2}{c}}}{\beta_1}} + \beta_2 y^{\frac{2}{c}-1}\ell^{-\frac{y^{\frac{2}{c}}}{\beta_1}}}{c\beta_1\left(\beta_1+\beta_2\right)}, y>0 \qquad (22)$$

Hence

$$M_{X^C}(t) = M_Y(t) = \frac{\left[\int_0^\infty y^{\frac{2}{c}-1}\ell^{-\frac{y^{\frac{2}{c}}}{\beta_1}}\ell^{ty}dy + \int_0^\infty y^{\frac{1}{c}-1}\ell^{-\frac{y^{\frac{1}{c}}}{\beta_1}} \times \ell^{ty}dy\right]}{c\beta_1\left(\beta_1+\beta_2\right)}$$

Let

$$v = \frac{y^{\frac{1}{c}}}{\beta_1}, therefore\ y = \left(\beta_1 v\right)^c, therefore\ dy = c\beta_1^c x v^{c-1}dv.$$

Therefore

$$M_Y(t) = \frac{1}{\beta_1\left(\beta_1+\beta_2\right)}\int_0^\infty \left(\beta_1^c v^c\right)^{\frac{2}{c}-1}\beta_1^c v^{c-1}\ell^{-v}.\ell^{t(\beta_1 v)^c}dv +$$

$$+\frac{\beta_2}{\beta_1\left(\beta_1+\beta_2\right)}\int_0^\infty \left(\beta_1^c v^c\right)^{\frac{1}{c}-1}\beta_1^c v^{c-1}\ell^{-v}.\ell^t\left(\beta_1 v\right)^c dv$$

$$M_X c\,(t) = M_y(t) = \frac{\beta_1}{\left(\beta_1+\beta_2\right)}\int_0^\infty v\left(1+\frac{t\left(\beta_1^c v^c\right)}{1!}+\frac{t^2\left(\beta_1^c v^c\right)^2}{2!}+....+\frac{t^n\left(\beta_1^c v^c\right)^n}{n!}\right)\ell^{-v}dv +$$

$$+\frac{\beta_2}{\left(\beta_1+\beta_2\right)}\int_0^\infty\left(1+\frac{t\left(\beta_1^c v^c\right)}{1!}+\frac{t^2\left(\beta_1^c v^c\right)^2}{2!}+....+\frac{t^n\left(\beta_1^c v^c\right)^n}{n!}........\right)\ell^{-v}dv$$

$$M_X c\,(t) = M_Y(t) = \frac{\beta_1\sum_{n=0}^\infty \frac{t^n\beta_1^{cn}\overline{)(cv+2)}}{n!} + \beta_2\sum_{n=0}^\infty \frac{t^n\beta_1^{cn}\overline{)(cv+1)}}{n!}}{\left(\beta_1+\beta_2\right)} \qquad (23)$$

The nth moment of the distribution of $Y = X^c$ about zero is taken as the co-efficient of $\frac{t^n}{n!}$ or the nth derivative of $M_Y(t)$ with respect to t evaluated at t = 0, in the expansion of Equation 23. That is,

$$M_Y(0)^{(n)} = M_{X^c}(0)^{(n)} = \frac{\beta_1^{cn}\left(\beta_1\overline{)(cn}+2)+\beta_2\overline{)(cn}+1)\right)}{\left(\beta_1+\beta_2\right)} = \frac{cn\overline{)(cn)}\beta_1^{cn}\left(\left(cn+1\right)\beta_1+\beta_2\right)}{\left(\beta_1+\beta_2\right)} \qquad (24)$$

This then yields the same result as our $\mu_{xn}(c)$ in Equation 19. Thus, the extra work involved in correctly obtaining the required moment using moment generating functions, when c, d or both are not whole numbers is quite lengthy. As may be easily verified, the above results, in cases where both c and d are whole numbers, are the same as would be obtained using the moment generating function for the distribution in Equation 17 namely:

$$M_{(x,y)}(t_1,t_2) = \frac{\left(\beta_1+\beta_2-\beta_1\beta_2(t_1+t_2)\left(1-\beta_1 t_1\right)^{-2}(1-\beta_2 t_2)^{-2}\right.}{\beta_1+\beta_2}$$

This is therefore the moment generating function of the joint probability density function of the random variables X and Y as given in Equation 17. However, in situations where c and d are not whole numbers, this moment generating function unlike the $\mu_n(c,d)$ cannot be correctly used without proper modifications to obtain the required moments of the distributions unless both cn and dn happen to be whole numbers. Unlike such other methods as the moment generating function, the $\mu_n(c,d)$ is able to yield moments of the distribution of all non-negative real powers of the random variables X and Y not just integer values, as in the case of moment generating function the moment of the marginal distribution of $X^c or\ Y^d$ are obtained by setting either d = 0 or c = 0 in their joint moment of powers of generating function $\left((mpgf)(\mu_n(c,d)\right)$. The proposed method $\mu_n(c,d)$ is easy to use since the process simply involves using the desired n in available formula and evaluating the results to obtain the required moment. Hence, in all cases $\mu_n(c,d)$ which always exists for all continuous distributions, is easier and quicker to use since the process simply involves using a desired n in the available formula and evaluating the result to obtain the required moment but in other methods such as factorial moments and moment generating function the process would involve first, properly modify the functions to be consistent with the desired distributions before differentiating the function, if it exists, n times before evaluating the results to obtain the required moment. This process clearly becomes increasingly more difficult and time consuming as n becomes larger.

CONCLUSIONS

We have presented in this paper the so called $\mu_n(c,d)$ as the nth moment of the joint distribution of the cth power of X and dth power of Y about zero for all non-negative values of c and d which may here be termed "moment of power generating function (mpgf), since it generates moments of the bi-variate distributions of powers of random variables X and Y. The proposed $\mu_n(c,d)$ exists for all continuous probability distributions unlike some of its competitors such as factorial moments of moment generating function which do not always exist. The results obtained using $\mu_n(c,d)$ are the same as results obtained using such other methods as moment generating functions of available. The proposed method is available and easy to use without the need for any modifications even when the powers of the random variable being considered are non-negative real numbers that do not need to be integers.

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