Physical Sciences Research International
Vol. 1(4), pp. 133-140, November 2013
Full Length Research Paper

Computational approach to cost and profit analysis of k-out of-n repairable system integrating human error and system failure constraints

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Accepted 8 November, 2013

ABSTRACT

This paper envisages a computational approach to demonstrate cost and profit analysis of k-out of–n repairable system taking into account the human error and common-cause system failure constraints. Earlier research work in queuing networks with system repair model boosts to confine attention for cost and profit analysis of k-out of–n repairable system due to its significant scope in pricing in modern-economics and informatics era, reliability engineering and system safety. In this paper, formulation of cost and profit functions of the model both in steady and transient states have been proposed. Moreover, results with numerical illustration have also been explored results for three different cases. Computed results for profit of the system both in steady and transient states in different three cases are shown in Tables 1 and 2. In addition, relevant graphs showing variation of profit with respect to time have also been displayed in Figures 1 and 2 particularly for sensitivity analysis and observational conclusions. By the end of paper, valuable discussions and significant conclusive observations have been presented.

Keywords: Cost analysis, profit function, mathematical modeling, repairable system, human error, common-cause system failure, steady state availability, Laplace transform, performance analysis.

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INTRODUCTION

Literature shows that cost and profit analysis of vide range of queuing network systems has attracted considerable attention of a large number of previous noteworthy researchers in modern-economics and informatics era and it has occupied a prominent place in Operational Research due to its significant value and application aspect for optimization of cost and profit function. Although, cost and profit analysis of queuing systems has received substantial attention by researchers since its origin but it has got more attention since last decade. For example, a number of previous researchers (Maurya and Maurya, 2013; Maurya, 2009; Mishra, 2006; Mishra and Yadav, 2008; Kumar et al., 2012) confined their attention in this direction. Keeping in view the demanding trend of cost and profit analysis of different versions of queuing systems since last decade, Maurya (2000) contributed for cost analysis of different version of Poisson queuing models in his doctoral thesis. Thereafter, Mishra (2006) focused his keen attention on the cost analysis of renewal model in clocked queuing network. Moreover, Maurya (2009) analyzed on optimality aspects of a generalized $M^X/E_K/1/\infty$ queuing model in transient conditions. In this paper, a computational approach to cost and profit analysis of k-out of-n repairable system involving human and system failure constraints has been presented. Here, it is remarked that some previous noteworthy researchers have contributed their devotion to analyze system repair model (Kee, 1987; Hughes, 1987; Madhu et al., 2002; Moustafa, 1997; Pham et al., 1997; Shao and Lamberson, 1991; Wang and Ming, 1997), yet very few researchers paid their attention to attempt for cost and profit analysis of system repair model and its versions (Maurya and Maurya, 2013; Maurya, 2009; Mishra, 2006; Mishra and Yadav, 2008; Kumar et al., 2012; Wang and Ming, 1997). Although, earlier research workers (Wang and Ming, 1997) focused their attention to attempt profit analysis of $M/E_K/1$ machine repair problem with a non-reliable service station but no attempt has still been made for evaluating cost and profit of k-out of-n repairable system due to human and common-cause system failures; which motivates us to investigate in this direction.

In a series systems, the failure of one or more units result in the system failure. However, there exist systems that are not considered failed until at least k units or components have failed. Such systems are known as k-out of-n systems. Examples of such systems are: large airplanes usually have three or four engines, but two engines may be the minimum number required to provide a safe journey. Similarly, in many power-generation systems that have two or more generators, one generator may be sufficient to provide the power requirements; for more details in this context, we refer Maurya (2013).

ASSUMPTIONS AND POSTULATES OF THE MODEL

In order to serve our objectives in the present paper, we use the following assumptions and postulates of the system repair model taken into consideration:

(i) The system consists of n identical main units and s standby units.
(ii) Unit failure, human error and common-cause failure are constant.
(iii) Repair rate from failed states due to unit failure, human error and common-cause failure are generally distributed.
(iv) Initially n units are operating and s unit are kept as cold standby.
(v) The entire system working if at least k of its units or components is operating.
(vi) The system is said to be in one of the failed if k+1 unit have failed due to unit failure, human error and common-cause failure.
(vii) When any of the operating units fails, it is replaced by standby unit.
(viii) If all the standbys are consumed, the system works as degraded system until k-units works.
(ix) No repair will be undertaken until the system has failed due to hardware and human error (that is, until k+1 unit have failed) or due to common-cause failure.
(x) A repaired system is as good as new.
(xi) A perfect switch is used to switch-on the standby units and switch-over time is negligible.

NOTATIONS AND BOUNDARY CONDITIONS OF THE MODEL

$P_{i,j}(t)$ Probability that the system is in state (i, j) at time t. State (i, j) is the state of the system when i units failed due to hardware failure and j units due to human error, i, j = 0, 1, 2, …, k+1. State (0, 0) is the initial state at t = 0 and states (i, k + 1 –i, i = 0, 1, …, k +1 are the failed states of the system. State cc is when the system has failed due to common-cause failure.
λ; constant hardware failure rate of a unit in the system
h; constant human error rate of a unit
λ_c; common-cause system failure rate
Λ = n λ, H = n h
ρ; Repair rate when the system is in state (i, j)
ρ_c ; Repair rate when the system is in state cc
$P_{i,j}(y,t)$; Probability density function that the failed system is in state (i, j) and has an elapsed repair time of y at time 't', j = k +1 –I, i= 0, 1, 2, …k +1
$P_{cc}(y,t)$; Probability density function that the failed system is in state cc and has an elapsed repair time of y at time t
s. Laplace transform variable, $\bar{F}(s)$ Laplace transform of the pdff(.)
$P_{up}(t)$; Operational time
$\mu_{up}(t)$; The expected operational time
$B(t)$; Busy period of the system
$\mu_B(t)$; The expected busy period of the system
B(∞); Steady state busy period
P_f(t); Profit function
P_f(∞); Profit function in steady state

Boundary conditions

$$P_{i,k+1-i}(0,t) = \Lambda P_{i-1,j}(t) + HP_{i,j-1}(t); i = 0,1,2,\ldots k+1 \quad (1)$$

$$P_{cc}(0,t) = \lambda_c \sum_{i=0}^{k} \sum_{j=0}^{k-i} P_{i,j}(t) \quad (2)$$

$$\sum_{i=0}^{k+1} \sum_{j=0}^{k+1-i} P_{i,j}(t) + P_{cc}(y,t) = 1 \quad (3)$$

Initial conditions

$$P_{0,,0}(t) = 1, P_{i,j}(t) = 0, i,j = 0,1,2,\ldots k+1 \quad (4)$$

PRELIMINARY IDEAS OF THE MODEL

Under assumptions and boundary conditions as used earlier, recently Maurya (2013) confined his attention to explore various significant results for k-out of-n repairable system taking into account the human error and system failure constraints and succeeded to investigate case wise following results among the others.

Case I

When n = 2, s = 1, k = 1, ρ, ρ_c are the repair rates when system is in state (i, j), and in cc states respectively.

$$P_{up}(t) = k_4 e^{r_1 t} + k_5 e^{r_2 t} + k_6 e^{r_3 t} + k_7 e^{r_4 t} \quad (5)$$

where

$$k_4 = \rho\rho_c(4(\lambda + h) + \lambda_c))/(-r_1r_2r_3)$$
$$k_5 = ((r_1 + \rho)(r_1 + \rho_c)(r_1 + 4(\lambda + h) + \lambda_c))/(r_1(r_1 - r_2)(r_1 - r_3))$$
$$k_6 = ((r_2 + \rho)(r_2 + \rho_c)(r_2 + 4(\lambda + h) + \lambda_c))/(r_2(r_2 - r_1)(r_2 - r_3))$$
$$k_7 = ((r_3 + \rho)(r_3 + \rho_c)(r_3 + 4(\lambda + h) + \lambda_c))/(r_3(r_3 - r_1)(r_3 - r_2))$$

and r_1, r_2, r_3 are the roots of the equation $s^3 + k^1s^2 + k^2s + k^3s = 0$

$$\mu_{up}(t) = \int_0^{\infty} P_{up}(t)\,dt = \frac{k_4}{r_1}(e^{r_1t} - 1) + \frac{k_5}{r_2}(e^{r_2t} - 1) + \frac{k_6}{r_3}(e^{r_3t} - 1) + \frac{k_7}{r_4}(e^{r_4t} - 1) \quad (6)$$

$$B(t) = k_4^1e^{r_1t} + k_5^1e^{r_2t} + k_6^1e^{r_3t} + k_7^1e^{r_4t} \quad (7)$$

where

$$k_4^1 = (\lambda_c\rho(4(h + \lambda) + \lambda_c) + 4\rho_c(h + \lambda)^2)/(-r_1r_2r_3)$$
$$k_5^1 = (\lambda_c(r_1 + 4(h + \lambda) + \lambda_c)(r_1 + \rho) + 4(h + \lambda)^2(r_1 + \rho_c))/(r_1(r_1 - r_2)(r_1 - r_3))$$
$$k_6^1 = (\lambda_c(r_2 + 4(h + \lambda) + \lambda_c)(r_2 + \rho) + 4(h + \lambda)^2(r_2 + \rho_c))/(r_2(r_2 - r_1)(r_2 - r_3))$$
$$k_7^1 = (\lambda_c(r_3 + 4(h + \lambda) + \lambda_c)(r_3 + \rho) + 4(h + \lambda)^2(r_3 + \rho_c)$$

$$\mu_B(t) = \int_0^{\infty} B(t)\,dt = \frac{k_4^1}{r_1}(e^{r_1t} - 1) + \frac{k_5^1}{r_2}(e^{r_2t} - 1) + \frac{k_6^1}{r_3}(e^{r_3t} - 1) + \frac{k_7^1}{r_4}(e^{r_4t} - 1) \quad (8)$$

$$B(\infty) = (\lambda_c(s + 4(h + \lambda) + \lambda_c)(s + \rho) + 4(h + \lambda)^2(s + \rho_c))/C'(0) \quad (9)$$

Case II

When n = 2, s = 1, k = 1, ρ are the repair occurs when system is in state (i, j).

$$P_{up}(t) = k_4 + k_5e^{r_1t} + k_6e^{r_2t} + k_7e^{r_3t} \quad (10)$$

where

$$k_4 = ((r_1 + \rho)(r_1 + 4(\lambda + h) + \lambda_c))/((r_1 - r_2)(r_1 - r_3)))$$
$$k_5 = ((r_2 + \rho)(r_2 + 4(\lambda + h) + \lambda_c))/((r_2 - r_1)(r_2 - r_3)))$$
$$k_6 = ((r_3 + \rho)(r_3 + 4(\lambda + h) + \lambda_c))/((r_3 - r_1)(r_3 - r_2)))$$

and r_1, r_2, r_3 are the roots of the equation $s^3 + k_1s^2 + k_2s + k_3 = 0$

$$\mu_{up}(t) = \int_0^{\infty} P_{up}(t)dt = k_4 + \frac{k_5}{r_1}(e^{r_1t} - 1) + \frac{k_6}{r_2}(e^{r_2t} - 1) + \frac{k_7}{r_3}(e^{r_3t} - 1) \quad (11)$$

$$B(t) = k_4^1e^{r_1t} + k_5^1e^{r_2t} + k_6^1e^{r_3t} \quad (12)$$

Where

$$k_4^1 = 4(h + \lambda)^2/(r_1 - r_2)(r_1 - r_3)$$
$$k_5^1 = 4(h + \lambda)^2/((r_2 - r_1)(r_2 - r_3)$$
$$k_6^1 = 4(h + \lambda)^2/((r_3 - r_1)(r_3 - r_2)$$

$$\mu_B(t) = \int_0^{\infty} B(t) = \frac{k_4^1}{r_1}(e^{r_1t} - 1) + \frac{k_5^1}{r_2}(e^{r_2t} - 1) + \frac{k_6^1}{r_3}(e^{r_3t} - 1) \quad (13)$$

$P_{up}(t) = \rho(4(\lambda + h) + \lambda_c)/(2(h + \lambda) + \lambda_c)^2 + 2\,\rho(2(h + \lambda) + \lambda_c$ (14)

$B(\infty) = (4(h + \lambda)^2)/(2(h + \lambda) + \lambda_c)^2 + 2\,\rho(2(h + \lambda) + \lambda_c)$ (15)

Case III

When n = 2, s = 1, k = 1, and no repair

$$P_{up}(t) = k_3 e^{r_1 t} + k_4 e^{r_2 t} \quad (16)$$

where

$$k_3 = (4(\lambda + h) + \lambda_c)/(r_1 - r_2)$$
$$k_4 = (4(\lambda + h) + \lambda_c))/(r_2 - r_1)$$

and r_1, r_2 are the roots of the equation $s2 + k_1 s + k_2 = 0$

$\mu_{up}(t) = \int_0^\infty P_{up}(t)\,dt = \frac{k_3}{r_1}(e^{r_1 t} - 1) + \frac{k_4}{r_2}(e^{r_2 t} - 1)$ (17)

$P_{up} = (4(\lambda + h) + \lambda_c)/(2(h + \lambda) + \lambda_c)^2)$ (18)

FORMULATION OF COST AND PROFIT FUNCTIONS OF THE MODEL

Case I

When n = 2, s = 1, k = 1, ρ, ρ_c are the repair rates when system is in state (i, j), and in cc states respectively.

Case II

When n = 2, s = 1, k = 1, ρ are the repair occurs when system is in state (i, j).

In both cases I and II, the expected total profit per unit time incurred to the system is given by the difference of total revenue and total cost. Hence, we have profit function as following:

$P_f(t) = R\mu_{up}(t) - C\mu_B(t)$ (19)

Similarly, the expected total profit per unit time incurred to the system in the steady-state is given by following equation:

$P_f(\infty) = RP_{up} - CB(\infty)$ (20)

where,

R: is the revenue per unit up-time of the system,
C : is the cost per unit time which the system is under repair

We remark here that Equations 19 and 20 are formula wise same in both cases however, their values will be different depending on case wise values of variation parameters of $\mu_{up}(t), \mu_B(t), P_{up}$ and $B(\infty)$.

Case III

When n = 2, s = 1, k = 1, and no repair

The expected total profit per unit time incurred to the system is given by the difference of total revenue and total cost. Since in this case, total cost of the system is zero because of no repair of units. Hence, we have profit function as following:

$$P_f(\mathrm{t}) = \mathrm{R}\mu_{up}(t) \tag{21}$$

And the expected total profit per unit time incurred to the system in the steady-state is given by:

$$P_f(\infty) = \mathrm{R}P_{up} \tag{22}$$

RESULTS

Setting values of parameters $\lambda = .02, h = .01, \lambda_c = .002, \rho = .8, \rho_c = .008$, one can compute

Case I

When n = 2, s = 1, k = 1, ρ, ρ_c are the repair rates when system is in state (i, j), and in cc states respectively.

$$\mu_{up}(t) = 0.84461t - 0.33687(\exp(-0.12737t) - 1)$$
$$-13.061(\exp(-9.1338 \times 10^{-3}t) - 1)$$
$$+8.5853 \times 10^{-3}(exp(-0.79463t) - 1) \tag{23}$$

$$\mu_B(t) = 0.24231t - 8.5785 \times 10^{-3}(\exp(-0.79463t) - 1)$$
$$+22.514(\exp(-9.1338 \times 10^{-3}t) - 1)$$
$$+0.34140(exp(-0.12737t) - 1) \tag{24}$$

$$P_f(\mathrm{t}) = \mathrm{R}\mu_{up}(t) - \mathrm{C}\,\mu_B(t) \tag{25}$$

where $\mu_{up}(t)$ and $\mu(t)$ functions are given in Equations 6 and 8

Case II

When n = 2, s = 1, k = 1, ρ are the repair occurs when system is in state (i, j).

$$\mu_{up}(t) = 8.5782 \times 10^{-3}(\exp(-0.79464t) - 1) - 0.34226\,(\exp(-0.12743t) - 1)$$
$$-499.66(exp(-1.9277 \times 10^{-3}t) - 1) \tag{26}$$

$$\mu_B(t) = -8.5660 \times 10^{-3}(\exp(-0.79463t) - 1) + 0.33734(\exp(-0.12744t) - 1)$$
$$-18.771(exp(-1.9276 \times 10^{-3}t) - 1) \tag{27}$$

$$P_f(\mathrm{t}) = \mathrm{R}\mu_{up}(t) - \mathrm{C}\mu_B(t) \tag{28}$$

In light of Equations 26 and 27, $P_f(\mathrm{t})$ from Equation 28 has been computed for varying time t.

Case III

When n = 2, s = 1, k = 1, and no repair

$$\mu_{up}\;(t) = 32.258(1 - exp\{-0.062t\}) - texp\{-0.062t\} \tag{29}$$

Table 1. Profit of the system for three different cases at time t.

t	Profit of the system with ρ and ρ_c	Profit of the system with ρ	Profit of the system without repair
0	0.0	0.0	0.0
10	979.81	976.53	952.55
20	1929.7	1916.3	1713.5
30	2863.5	2832.9	2256.6
40	3786.1	3730.4	2620.7
50	4699.4	4610.5	2855.2
60	5604.4	5473.6	3002.2
70	6502.2	6320.2	3092.5
80	7393.0	7150.7	3147.1
90	8277.7	7965.2	3179.7
100	9156.9	8764.2	3199.0

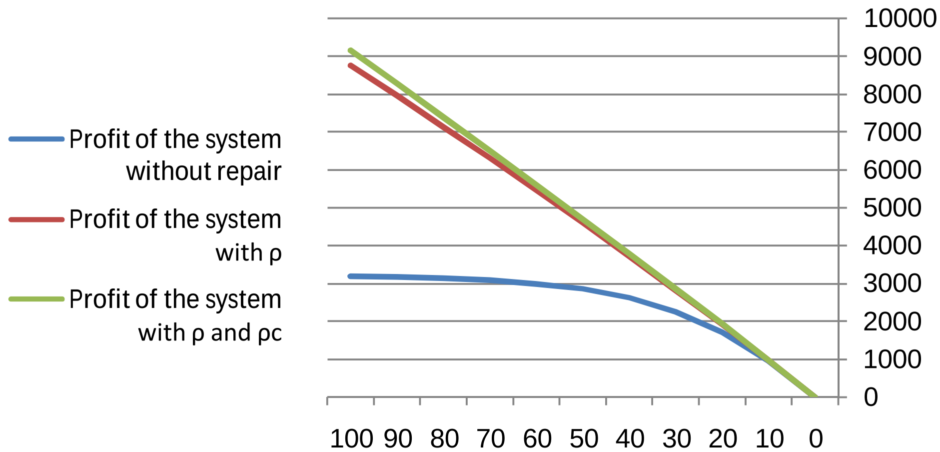

Figure 1. Variation of profit w.r.t. time in the three cases.

Table 2. Steady state profit of the system for different cases.

h	Steady state profit of the system with ρ and ρ_c	Steady state profit of the system with ρ
0.0	76.316	94.89
0.02	74.656	93.501
0.04	73.058	91.469
0.06	71.519	89.348
0.08	70.036	87.252
0.10	68.606	85.211
0.12	67.227	83.237
0.14	65.895	81.333
0.16	64.608	79.499
0.18	63.363	77.733
0.20	62.161	76.032

$$P_f(\mathrm{t}) = \mathrm{R}\mu_{up}(t) \tag{30}$$

Setting t = 0, 1, 2, ..., in Equations 28, 29 and 30, one can get Table 1. Variation of profit w.r.t. time in three cases is shown in Figure 1.

Setting h = 0, .02, .04, ..., in Equations 14, 15 and 18, it is fairly easy to get Table 2 from equations 20 to 22. Variation of steady state profit for different values of human error in three cases is shown in Figure 2.

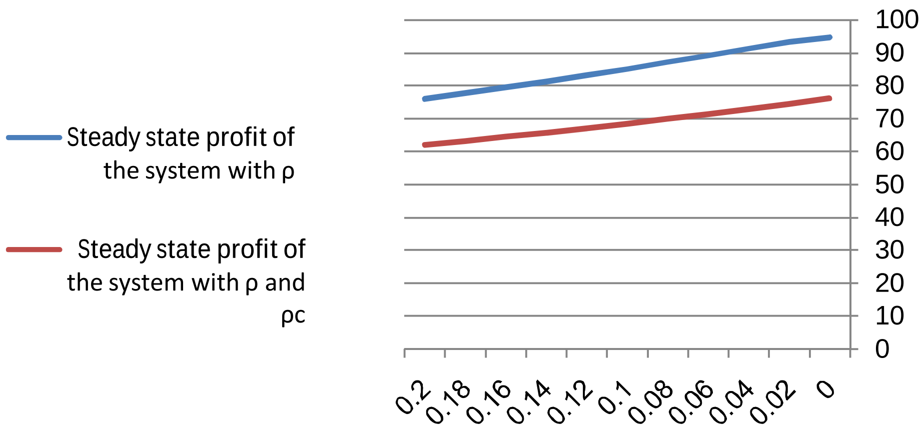

Figure 2. Variation of steady state profit for different values of human error for cases II and III.

DISCUSSION AND CONCLUSION

Here, we focus our attention on the sensitivity analysis of one parameter relative to other parameters for determining the trend of future data-input. Some significant observations on the basis of graphs demonstrated in Figures 1 and 2.

Table 1 computes profit function of the system at any time and graph in Figure 1 shows expected total gain increases in the interval (0, t) for the three cases. By comparing the profit function with respect to time t for three cases with and without repair graphically, it has been observed that it increases with respect to time t.

The system profit with ρ and ρc is greater than the system with ρ and profit with ρ repair is greater than the system without repair.

Profit of the system without repair is a non-linear function which initially increases rapidly with respect to time but it increases slowly after a certain time (t = 60).

Both profit of the system with ρ only that of with ρ and ρc are linear functions and profit of the system in both cases increases with respect to time. However, profit of the system with ρ and ρc is higher to that of with ρ only; which shows evidently that common-cause failure constraint affects the profit of the system.

Table 2 computes variation of steady-state profit with respect to human failure. The graph in Figure 2 shows variation of steady state profit for different values of human error for cases II and III.

By comparing the steady state profit with respect to human failure for the systems with ρ and with ρ, ρc graphically, it has been critically observed that the increasing value of human failure rate hat constant $\lambda = .02, \lambda_c = .002, \rho = .8, \rho_c = .008$ results decreasing pattern of profit of the system in steady state.

In steady state, the profit of system with repair ρ only is greater than the system with ρ and ρc (that is, system includes common-cause).

Finally with passing above remarks, we further conclude that the results explored in the present paper are quite useful for researchers, engineers and experts dealing with problems of machine repair problems such as in field of reliability and industrial engineering. Particularly, cost and profit analysis of system repair problem with human error and common-cause failures reflect its significant application in pricing in modern-economics and informatics era as well as in quality control also.

ACKNOWLEDGEMENTS

The author would like to express his heartiest gratitude to Honorable Vice Chancellor, University of Fiji, Saweni and Dean, School of Science and Technology, University of Fiji, Saweni for their continuous encouragement and sincere support to provide all essential facilities and his D.Sc. Advisor Prof. (Dr.) Ram Bilas Misra, Ex Vice Chancellor, Dr. Ram Manohar Lohia Avadh University, Faizabad, U.P., India.

REFERENCES

Hughes RP, 1987. A new approach to common-cause failure. Reliab Eng Syst Safety, 17:211-236.

Kee CW, 1987. Reliability analysis of a repairable parallel system with standby involving human failure and common-cause failures, Microelect Reliab, 27(2):269-271.

Kumar R, Maurya VN, Maurya AK, 2012. A cost- benefit model for evaluating regression testing technique. Int J Software Eng Comput, 4(2):84-89.

Madhu J, Sharma GC, Kumar A, 2002. K-out of-n system with dependent failure and standby support. JKAU: Eng Sci, 14(2):73-83.

Maurya AK, Maurya VN, 2013. A novel algorithm for optimum balancing energy consumption LEACH protocol using numerical simulation technique. Int J Elect Commun Elect Eng, 3(4):1-19.

Maurya VN, 2009. On optimality aspects of a generalized $M^X/E_K/1/\infty$ queuing model – Transient approach. Int J Eng Res Ind Appl. 2(4):355-368.

Mishra SS, 2006. Cost analysis of renewal model in clocked queuing networks. SCRA 2006-*FIM* 13th International Conference on Interdisciplinary Mathematical and Statistical Techniques, USA.

Mishra SS, Yadav DK, 2008. Cost and profit analysis of $M/E_K/1$ queuing system with removable service station. Bulgarian J Appl Math Sci,

2(56):2777-2784.
Moustafa SM, 1997. Reliability analysis of k-out of-n systems with dependent failures and imperfect coverage. Reliab Eng Syst Safety, 58:15-17.
Pham H, Suprasad A, Misra RB, 1997. Availability and mean life time predication of multistage degraded system with partial repair. Reliab Eng Syst Safety, 56:169-173.
Shao J, Lamberson LR, 1991. Modeling a shared-load k-out of-n: G systems, IEEE Trans Reliab, 40(2):202-208.
Wang KH, Ming Y, 1997. Profit analysis of $M/E_K/1$ machine repair problem with a non-reliable service station. Comput Ind Eng, 32:587-594.